AQA Maths Mechanics 3

Mark Scheme Pack

2006-2015

Version 1.0: 0606

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MM03 Mechanics 3

Mark Scheme

2006 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1(a)(i)	$\mathrm{T}^1 = \mathrm{L}^a \times \mathrm{M}^b \times (\mathrm{LT}^{-2})^c$	M1A1		
	There is no M on the left, so $b = 0$	E1	3	
(ii)	$\mathrm{T}^1 = \mathrm{L}^{a+c} \times \mathrm{M}^0 \times \mathrm{T}^{-2}$	M1		
	$\begin{cases} -2c = 1 \\ a + c = 0 \end{cases}$	m1		equating corresponding indices
	$a = \frac{1}{2},\ c = -\frac{1}{2}$	m1		solution
	$\therefore$ Period $= kl^{\frac{1}{2}} g^{-\frac{1}{2}}$	A1F	4	constant needed
	Total		**7**	
2(a)	conservation of momentum			
	$mu = mv_A + mv_B$	M1		
	$u = v_A + v_B$	A1		
	restitution			
	$eu = v_B - v_A$	M1A1		OE
	$v_B = \frac{1}{2}u(1+e)$	A1F	5	OE
(b)	$mv_B = mw_B + 2m\frac{3u}{8}$	M1A1		
	$ev_B = \frac{3u}{8} - w_B$	M1A1		OE
	Elimination of w_B	m1		dependent on both M1s
	$4e^2 + 8e - 5 = 0$	A1F		simplified quadratic equation in e only
	$e = \frac{1}{2}$	A1F	7	stated as the only value $(0 < e < 1$ for follow through$)$
	Total		**12**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$I = 1.4\times10^5 \int_0^{0.1} (t^2 - 10t^3)\mathrm{d}t$	M1A1		
	$= 1.4\times10^5 \left[\frac{1}{3}t^3 - \frac{10}{4}t^4\right]_0^{0.1}$	m1		
	$= 11.7$ Ns	A1	4	AG
(b)	initial momentum $= 0.45(-15)$ $= -6.75$ Ns	M1		
	final momentum $= 11.7 - 6.75$ $= 4.95$ Ns	M1		
	velocity after impact $= \frac{4.95}{0.45}$	m1		dependent on both previous M1s
	$= 11\ \mathrm{ms}^{-1}$	A1	4	
(c)	The ball is not perfectly elastic or $e \neq 1$ or energy loss	E1	1	
	Total		**9**	

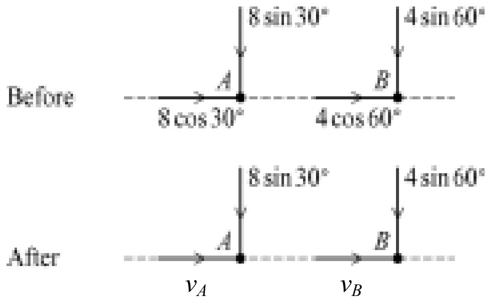
MM03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	${}_A\mathbf{v}_B = (12\mathbf{i} - 8\mathbf{j}) - (6\mathbf{i} + 12\mathbf{j})$	M1		
	$= 6\mathbf{i} - 20\mathbf{j}$	A1	2	needs to be in terms of **i** and **j**
(b)	${}_A\mathbf{r}_B = \mathbf{r}_0 + {}_A\mathbf{v}_B t$	M1A1		attempted use
	${}_A\mathbf{r}_B = (18\mathbf{i} + 5\mathbf{j}) - (5\mathbf{i} - \mathbf{j}) + (6\mathbf{i} - 20\mathbf{j})t$	A1F		
	${}_A\mathbf{r}_B = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$	A1	4	AG (not penalised if not in terms of **i** and **j**)
	Alternative			
	$\mathbf{r}_A = 5\mathbf{i} - \mathbf{j} + (6\mathbf{i} + 12\mathbf{j})t$			
	$\mathbf{r}_B = 18\mathbf{i} + 5\mathbf{j} + (12\mathbf{i} - 8\mathbf{j})t$	M1A1		A1 for each of $\mathbf{r}_A$ and $\mathbf{r}_B$
	${}_A\mathbf{r}_B = 18\mathbf{i} + 5\mathbf{j} + (12\mathbf{i} - 8\mathbf{j})t - [5\mathbf{i} - \mathbf{j} + (6\mathbf{i} + 12\mathbf{j})t]$	A1		
	${}_A\mathbf{r}_B = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$	A1F		
(c)	$s^2 = (13 + 6t)^2 + (6 - 20t)^2$	M1A1F		attempt for squaring and tidying up
	A and B are closest when $\frac{\mathrm{d}s}{\mathrm{d}t} = 0$ or $\frac{\mathrm{d}s^2}{\mathrm{d}t} = 0$	M1		
	$2s\frac{\mathrm{d}s}{\mathrm{d}t} = 2(13 + 6t)6 - 2(6 - 20t)20 = 0$	M1 A1		accuracy of differentiation
	$t = 0.0963$ $\left(\text{or } 0.096 \text{ or } \frac{21}{218}\right)$	A1F	6	
	Alternative			
	${}_A\mathbf{r}_B \cdot {}_A\mathbf{v}_B = 0$	M1		
	$[(13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}] \cdot [6\mathbf{i} - 20\mathbf{j}] = 0$	M1		
	$6(13 + 6t) - 20(6 - 20t) = 0$	M1A1		
	$436t - 42 = 0$	A1F		
	$t = 0.0963$ (or 0.096 or $21/218$)	A1F		
(d)	$s = \sqrt{(13 + 6 \times 0.0963)^2 + (6 - 20 \times 0.0963)^2}$	m1		dependent on M1s in part (c)
	$s = 14.2$ km	A1F	2	AWRT
	Total		**14**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y=-\frac{1}{2}gt^2+20\sin 30.t$	M1A1		
	$x=20\cos 30.t$	M1		
	$t=\frac{x}{20\cos 30}$	A1		
	$y=-\frac{1}{2}g\frac{x^2}{400\cos^2 30}+20\sin 30\frac{x}{20\cos 30}$	M1		
	$y=x\tan 30-\frac{gx^2}{800\cos^2 30°}$	A1	6	AG
(b)	$2.5=x\tan 30-\frac{9.8x^2}{800\cos^2 30}$			
	$9.8x^2-346x+1500=0$	M1A1		substituting and tidying up
	$x=\frac{346\pm\sqrt{119716-58800}}{19.6}$	M1		
	=30.3 (or 30.2) & 5.06 (or 5.05)			
	answer: 30.3m (or 30.2m)	A1F	4	at least 3 s.f. required
(c)	no air resistance,	B1		
	the ball is a particle	B1	2	
	etc.			
	Total		**12**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Components of velocities:			
				
	conservation of linear momentum along the line of centres:			
	$m\times 8\cos 30 + m\times 4\cos 60 = mv_A + mv_B$	M1A1		OE unsimplified
	$v_A + v_B = 8.93$			
	Law of restitution along the line of centre:			
	$\dfrac{v_B - v_A}{8\cos 30 - 4\cos 60} = \dfrac{1}{2}$	M1A1		OE unsimplified
	$v_B - v_A = 2.46$			
	$v_B = 5.70$	m1		dependent on both M1s
		A1F		AWRT $\left(\text{or } 3\sqrt{3} + \dfrac{1}{2}\right)$
	momentum of B perpendicular to the line of centres is unchanged	B1		PI (can also be gained in part (b))
	Speed of $B = \sqrt{{u_B}^2 + {v_B}^2}$			
	$= \sqrt{(4\sin 60)^2 + (5.70)^2}$	m1		dependent on both M1s
	$= 6.67$	A1F	9	
(b)	direction of $B = \tan^{-1}\dfrac{4\sin 60}{5.70} = 31.3°$	m1		dependent on both M1s and B1
		A1F	2	
	Total		**11**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	the projectile hits the plane again when			
	$(Ut\sin\theta - \frac{1}{2}gt^2\cos\alpha) = 0$	M1A1		
	$\therefore t = \dfrac{2U\sin\theta}{g\cos\alpha}$	A1F	3	need to be simplified
(ii)	the component of velocity perpendicular to plane $=$			
	$U\sin\theta - g\dfrac{2U\sin\theta}{g\cos\alpha}\cos\alpha =$	M1A1F		
	$-U\sin\theta =$ the initial magnitude	A1	3	AG
(b)	Newton's law of restitution perpendicular to plane:			
	$u = eU\sin\theta$	M1		
	$a = -g\cos\alpha$			
	$s = 0$			
	$0 = eU\sin\theta.T - \frac{1}{2}g\cos\alpha.T^2$	M1 A1		
	$T = \dfrac{2eU\sin\theta}{g\cos\alpha} = et$			
	$t:T = 1:e$	A1F	4	
	Total		**10**	
	TOTAL		**75**	

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Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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1(a)	$\text{MLT}^{-2} = \dfrac{[G]\text{MM}}{\text{L}^2}$	M1 A1		
	$[G] = \text{L}^3\text{M}^{-1}\text{T}^{-2}$	A1F	3	
(b)	$t = km^{\alpha}R^{\beta}G^{\gamma}$			
	$\text{T} = \text{M}^{\alpha}\text{L}^{\beta}\text{M}^{-\gamma}\text{L}^{3\gamma}\text{T}^{-2\gamma}$	M1 A1F		L, M, T for *G* are needed to gain M1
	$-2\gamma = 1 \quad \Rightarrow \quad \gamma = -\dfrac{1}{2}$			
	$\alpha - \gamma = 0 \quad \Rightarrow \quad \alpha = -\dfrac{1}{2}$	m1 m1		Getting 3 equations Solution
	$\beta + 3\gamma = 0 \quad \Rightarrow \quad \beta = \dfrac{3}{2}$	A1F	5	Finding α, β, γ
	Total		**8**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
2 (a)	${}_B\mathbf{v}_A = \mathbf{v}_A - \mathbf{v}_B$ $= (20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k}) - (30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})$ $= -10\mathbf{i} - 20\mathbf{j} + 10\mathbf{k}$	M1A1	2	Simplification not necessary
(b)	${}_B\mathbf{r}_{0A} = (8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k})$ $-(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})$ $= 6000\mathbf{i} + 1000\mathbf{j} + 2000\mathbf{k}$	M1		
	${}_B\mathbf{r}_A = (6000\mathbf{i} + 1000\mathbf{j} + 2000\mathbf{k})$ $+(-10\mathbf{i} - 20\mathbf{j} + 10\mathbf{k})t$ ${}_B\mathbf{r}_A = (6000 - 10t)\mathbf{i} + (1000 - 20t)\mathbf{j}$ $+(2000 + 10t)\mathbf{k}$	M1 A1F	3	Simplification not necessary
(c)	$\lvert {}_B\mathbf{r}_A \rvert^2 = (6000 - 10t)^2 + (1000 - 20t)^2$ $+(2000 + 10t)^2$	M1 A1F		
	The helicopters are closest when $\lvert {}_B\mathbf{r}_A \rvert^2$ is minimum. $y = (6000 - 10t)^2 + (1000 - 20t)^2$ $+(2000 + 10t)^2$ $\frac{dy}{dt} = 2(-10)(6000 - 10t)$ $+2(-20)(1000 - 20t)$ $+ 2(10)(2000 + 10t) = 0$	m1 A1F		
	$t = 100$	A1F	5	
	Alternative: $\begin{pmatrix} 6000 - 10t \\ 1000 - 20t \\ 2000 + 10t \end{pmatrix} \bullet \begin{pmatrix} -10 \\ -20 \\ 10 \end{pmatrix} = 0$	(M1) (A1F)		
	$-60000 + 100t - 20000 + 400t$ $+20000 + 100t = 0$	(m1) (A1F)		
	$600t = 60000$ $t = 100$	(A1F)	(5)	
	Total		**10**	

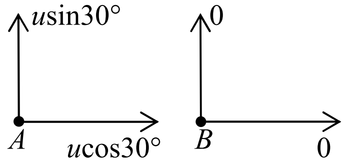
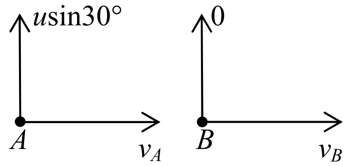
MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$I = \int_0^3 (4t+5)\mathrm{d}t$	M1		
	$= \left[2t^2 + 5t\right]_0^3$	m1		Or evaluation of constant
	$= 33$ Ns	A1	3	
	Alternative:			
	I = Area under the Force–Time graph	(M1)		
	$= \dfrac{17+5}{2} \times 3$	(m1)		
	$= 33$ Ns	(A1)	(3)	
(b)	$I = mv - mu$			
	$33 = 2v - 2(0)$	M1		
	$v = 16.5$ ms^{-1}	A1F	2	
(c)	$I = \int_0^t (4t+5)\mathrm{d}t = 2(37.5) - 2(0)$	M1		
	$2t^2 + 5t - 75 = 0$	A1		
	$t = \dfrac{-5 \pm \sqrt{25 + 8 \times 75}}{4}$	m1		
	$t = 5$	A1F	4	For one value of t identified only
	Total		**9**	
4(a)	Conservation of momentum :			
	$0.3(3) - 0.2(2) = 0.3v_A + 0.2v_B$	M1A1		
	$3v_A + 2v_B = 5$ ----------------(1)			
	Newton's experimental law :			
	$0.8 = \dfrac{v_B - v_A}{5}$	M1		
	$v_B - v_A = 4$ -----------------(2)	A1		For both (1) and (2)
	Solving (1) and (2)	m1		Dependent on both M1s
	$v_B = 3.4$			
	$v_A = -0.6$	A1F	6	For both solutions
(b)	$0.7 = \dfrac{v}{3.4}$	M1		
	$v = 2.38$	A1F		
	Speed of B (2.38) $\succ$ Speed of A (0.6)			
	$\therefore$ B collides again with A	E1	3	Cannot be gained without A1F
	Total		**9**	

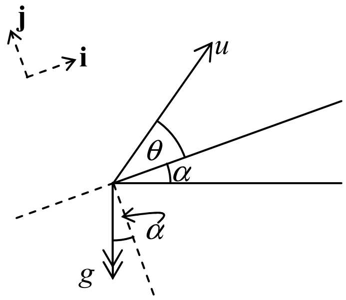
MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$y = ut\sin\alpha - \frac{1}{2}gt^2$	M1 A1		
	$x = ut\cos\alpha$	M1		
	$t = \frac{x}{u\cos\alpha}$	A1		
	$y = u\left(\frac{x}{u\cos\alpha}\right)\sin\alpha - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	M1		
	$y = x\tan\alpha - \frac{gx^2}{u^2\cos^2\alpha}$			
	$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$	A1	6	Answer given
(b)(i)	$1 = R\tan\alpha - \frac{10R^2}{2u^2}(1+\tan^2\alpha)$	M1		
	$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$	A1	2	Answer given
(ii)	For real solutions of the quadratic :			
	$u^4R^2 - 20R^2(5R^2 + u^2) \geq 0$	M1		
	$R^2 \leq \frac{u^4 - 20u^2}{100}$			
	$R^2 \leq \frac{u^2(u^2 - 20)}{100}$	A1	2	Answer given
(iii)	$5^2 \leq \frac{u^2(u^2 - 20)}{100}$			
	$u^4 - 20u^2 - 2500 \geq 0$	M1		Condone equation
	${u_{\min}}^2 = 61.0$ (or $10 + \sqrt{2600}$)	A1		
	$u_{\min} = 7.81$	A1F	3	3 sf required
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Before: 			
	After: 			
	Con. of Mom. along the line of centres:			
	$mu\cos 30° = mv_A + mv_B$	M1		
	$v_A + v_B = \frac{\sqrt{3}}{2}u$ ----------(1)	A1		
	Newton's experimental law :			
	$e = \frac{v_B - v_A}{u\cos 30° - 0}$	M1		
	$v_B - v_A = \frac{\sqrt{3}}{2}ue$ ----------(2)	A1		
	Solving (1) and (2) :			
	$v_B = \frac{\sqrt{3}}{4}u(1+e)$	A1	5	Answer given
(b)	$\perp$ $u\sin 30° = \frac{1}{2}u$	B1		$u$sin30 accepted
	$\parallel$ $v_A = \frac{\sqrt{3}}{2}u - \frac{\sqrt{3}}{4}u(1+e)$	M1 A1F	3	Simplification not needed
	$v_A = \frac{\sqrt{3}}{4}u(1-e)$			
(c)	$\alpha = \tan^{-1}\frac{\frac{1}{2}u}{\frac{\sqrt{3}}{4}u\left(1-\frac{2}{3}\right)}$	M1 A1F		
	$\alpha = \tan^{-1}\frac{6}{\sqrt{3}}$			
	$\alpha = 74°$	A1F	3	To the nearest degree required
	Total		**11**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)				
	$y = ut\sin\theta - \frac{1}{2}gt^2\cos\theta$	M1A1		
	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	A1F		
	$x = ut\cos\theta - \frac{1}{2}gt^2\sin\alpha$	M1A1		
	$R = u\frac{2u\sin\theta}{g\cos\alpha}\cos\theta - \frac{1}{2}g\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2\sin\alpha$	M1		
	$R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}$	m1 A1	8	Dependent on M1s Answer given
(b)	$R = \frac{2u^2 \times \frac{1}{2}[\sin(2\theta+\alpha)+\sin(-\alpha)]}{g\cos^2\alpha}$	B1		
	R is maximum when $\sin(2\theta+\alpha) = 1$	M1		
	or $2\theta + \alpha = \frac{\pi}{2}$			
	$\therefore \quad \theta = \frac{\pi}{4} - \frac{\alpha}{2}$	A1	3	Answer given
(c)	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	M1 A2,1		For using y=0 and $\dot{x} = 0$ A2 for both correct
	$\dot{x} = 0 \quad \Rightarrow \quad t = \frac{u\cos\theta}{g\sin\alpha}$			
	$\frac{2u\sin\theta}{g\cos\alpha} = \frac{u\cos\theta}{g\sin\alpha}$			
	$2\tan\theta = \cot\alpha$	A1	4	Answer given N.B. A problem arose which ultimately affected the marking of part 7(c). Please see the Report on the Examination for details.
	Total		**15**	
	TOTAL		**75**	

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Mark Scheme

2008 examination - June series

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Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX *Dr Michael Cresswell Director General*

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1	$LT^{-1} = L^{\alpha} \times (ML^{-3})^{\beta}(LT^{-2})^{\gamma}$	M1		
	There is no M on the left hand side, so $\beta = 0$.	E1		
	$LT^{-1} = L^{\alpha+\gamma}T^{-2\gamma}$	m1		Dependent on M1
	$\alpha + \gamma = 1$ $-2\gamma = -1$	m1		Equating corresponding indices
	$\gamma = \dfrac{1}{2}$	A1		
	$\alpha = \dfrac{1}{2}$	A1	6	
	Total		**6**	
2(a)	${}_Av_B = v_B - v_A$ $= (3i + 4j) - (5i - j)$ $= -2i + 5j$	M1 A1	2	
(b)	${}_Ar_{0B} = (40i - 90j) - (-60i + 160j)$ $= 100i - 250j$	M1		Simplification not necessary
	${}_Ar_B = (100i - 250j) + (-2i + 5j)\mathrm{t}$	m1 A1F	3	**ALTERNATIVE :** $r_A = (60i + 160j) + (5i - j)\mathrm{t}$ M1 $r_B = (40i - 90j) + (3i + 4j)\mathrm{t}$ ${}_Ar_B = \left[(40i - 90j) + (3i + 4j)\mathrm{t}\right] - \left[(60i + 160j) + (5i - j)\mathrm{t}\right]$ m1A1
(c)	${}_Ar_B = (100 - 2\mathrm{t})i + (-250 + 5t)\mathrm{j}$	M1		Collecting i and j terms
	$(100 - 2\mathrm{t}) = 0 \iff \mathrm{t} = 50$ $(-250 + 5\mathrm{t}) = 0 \iff \mathrm{t} = 50$	A1F		
	$\therefore$ A and B would collide.	E1	3	
			ALTERNATIVE : $\left[(100 - 2\mathrm{t})i + (-250 + 5\mathrm{t})j\right].(-2i + 5j) = 0$ M1 $-200 + 4\mathrm{t} - 1250 + 25\mathrm{t} = 0 \Rightarrow \mathrm{t} = 50$ A1 $\left\|{}_Ar_B\right\| \sqrt{(100 - 2\times50)^2 + (-250 + 5\times50)^2} = 0$ $\therefore$ A and B would collide E1	
	Total		**8**	

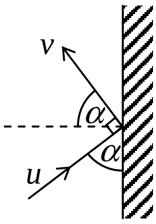
MM03 (cont)

Q	Solution	Marks	Total	Comments
3	$\int_0^t 5\times10^3 t^2 \,\mathrm{d}t = 0.2(2) - 0.2(0)$	M1A1		Impulse-Momentum principle
	$\frac{5\times10^3}{3}t^3 = 0.4$	A1F		
	$t = 0.0621$	A1F	4	At least 3 sig. fig. required
	Total		**4**	
4(a)	C.L.M. $m(4\mathbf{i} + 3\mathbf{j}) + 2m(-2\mathbf{i} + 2\mathbf{j}) = mv + 2m(\mathbf{i} + \mathbf{j})$ $7\mathbf{j} = v + (2\mathbf{i} + 2\mathbf{j})$ $v = -2\mathbf{i} + 5\mathbf{j}$	M1 A2,1,0	3	A1 for one slip
(b)	The angle with **j** direction : $A: \quad \tan^{-1}\frac{2}{5} = 21.8^\circ$ $B: \quad \tan^{-1}\frac{1}{1} = 45^\circ$ The angle $= 21.8^\circ + 45^\circ = 67^\circ$	M1 A1F	3	OE. in **i** direction M1 for two inverse tan and addition of angles AWRT. **Alternative (not in the specification)** $(-2\mathbf{i}+5\mathbf{j}).(\mathbf{i}+\mathbf{j}) = \sqrt{29}\times\sqrt{2}\cos\theta$ (M1) $\cos\theta = \frac{3}{\sqrt{58}}$ (A1) $\theta = 67^\circ$ (A1F) awrt
(c)	The impulse = Gain in momentum of A $= m(-2\mathbf{i} + 5\mathbf{j}) - m(4\mathbf{i} + 3\mathbf{j})$ $= -6m\mathbf{i} + 2m\mathbf{j}$	M1 A1F A1F	3	
(d)	$-3\mathbf{i} + \mathbf{j}$ or any scalar multiple of $-3\mathbf{i} + \mathbf{j}$	B1	1	
	Total		**10**	

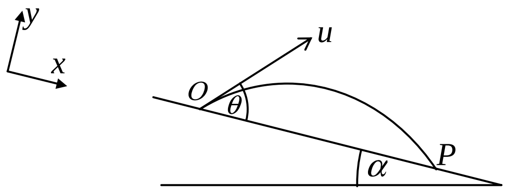
MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$5 = 10\cos\alpha.t$	M1		
	$t = \dfrac{5}{10\cos\alpha}$	A1		
	$1 = -\dfrac{1}{2}(9.8)t^2 + 10\sin\alpha.t$	M1A1		
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100\cos^2\alpha} + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	m1		Dependent on both M1s
	$1 = -\dfrac{1}{2}(9.8)\dfrac{25}{100}(1+\tan^2\alpha) + 10\sin\alpha\dfrac{5}{10\cos\alpha}$	A1		
	$49\tan^2\alpha - 200\tan\alpha + 89 = 0$	A1	7	Answer given
(b)	$\tan\alpha = \dfrac{200 \pm \sqrt{40000 - 4(49)(89)}}{2\times 49}$	M1		
	$= 3.57, 0.508$	A1		AWRT
	$\alpha = 74.4^\circ, 26.9^\circ$	A1F	3	
(c)(i)	$10\cos 26.9^\circ = 8.92$ (or 8.91) > 8			
	$\Rightarrow$ The can will be knocked off the wall	M1		Both values checked
	$10\cos 74.4^\circ = 2.69 < 8$	A1F		Acc. of both results
	$\Rightarrow$ The can will not be knocked off the wall	E1	3	Correct conclusions
		ALTERNATIVE The can will be knocked off the wall if $10\cos\alpha > 8$ $\cos\alpha > 0.8$ $\alpha < 36.9^\circ$ M1A1 So, for $\alpha = 26.9^\circ$ the can will be knocked off and for $\alpha = 74.4^\circ$, the can will not be knocked off E1		
5(c)(ii)	$x = ut$			
	$t = \dfrac{5}{10\cos 26.9^\circ}$			
	$v = 10\sin 26.9^\circ - 9.8\left(\dfrac{5}{10\cos 26.9^\circ}\right)$	M1		Any correct use of equations
	$v = -0.970$	A1F		
	$\tan\theta = \dfrac{-0.970}{8.92}$	M1		
	$\theta = -6.2^\circ$			
	At an angle of depression of 6.2°	A1F	4	AWRT 6°
	Total		**17**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
6(a)				
	Parallel to the wall : velocity is unchanged $u\cos\alpha = v\sin\alpha$	M1		
	Perpendicular to the wall : Law of Restitution $\frac{v\cos\alpha}{u\sin\alpha} = \frac{3}{4}$	M1		
	$\frac{v\cos\alpha}{v\tan\alpha\sin\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\frac{\cos^2\alpha}{\sin^2\alpha} = \frac{3}{4}$	m1		Dependent on both M1s
	$\tan^2\alpha = \frac{4}{3}$			
	$\tan\alpha = \frac{2}{\sqrt{3}}$	A1	5	Answer given
(b)	$v = \frac{u}{\tan\alpha}$	M1		
	$v = \frac{\sqrt{3}}{2}u$ or $0.866u$	A1	2	
(c)	Magnitude of Impulse = Change in momentum perpendicular to the wall	M1		
	$= 0.2\times v\cos\alpha - (-0.2\times 4\sin\alpha)$	A1 A1		
	$= 0.2\times\frac{\sqrt{3}}{2}\times 4\cos\alpha + 0.2\times 4\sin\alpha$	m1		
	= 1.06 Ns	A1F		
	Average Force = $\frac{1.06}{0.1} = 10.6$ N	A1F	6	
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7				
(a)	$v_y{}^2 = u^2\sin^2\theta - 2g\cos\alpha . y$	M1 A1		
	$0 = u^2\sin^2\theta - 2g\cos\alpha . y_{\max}$	m1		
	$y_{\max} = \dfrac{u^2\sin^2\theta}{2g\cos\alpha}$	A1F	4	
(b)(i)	$u\sin\theta\, t - \frac{1}{2}g\cos(\alpha)t^2 = 0$	M1		
	$t = \dfrac{2u\sin\theta}{g\cos\alpha}$	A1	2	
(ii)	$x = u\cos\theta t - \frac{1}{2}g\sin(-\alpha)t^2$	M1 A1		
	$R = u\cos\theta(\dfrac{2u\sin\theta}{g\cos\alpha}) + \dfrac{1}{2}g\sin\alpha(\dfrac{2u\sin\theta}{g\cos\alpha})^2$	M1		
	$= \dfrac{2u^2\cos\theta\sin\theta\cos\alpha + 2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$	m1		Dependent on both M1s
	$= \dfrac{2u^2\sin\theta(\cos\theta\cos\alpha + \sin\theta\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$= \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$	A1	6	Answer given
(iii)	$\overline{OP} = \dfrac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$			
	$= \dfrac{2u^2\frac{1}{2}[\sin(2\theta-\alpha)+\sin\alpha]}{g\cos^2\alpha}$	M1A1		
	$\overline{OP}$ is max when $\sin(2\theta-\alpha) = 1$	M1		
	$\overline{OP}_{\max} = \dfrac{u^2(1+\sin\alpha)}{g\cos^2\alpha}$	A1F		
	$\overline{OP}_{\max} = \dfrac{u^2(1+\sin\alpha)}{g(1-\sin^2\alpha)}$			
	$\overline{OP}_{\max} = \dfrac{u^2}{g(1-\sin\alpha)}$	A1	5	Answer given
	Total		**17**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	**ALTERNATIVE**			
	$0 = u\sin\theta - g\cos a\ t$	M1		
	$t = \dfrac{u\sin\theta}{g\cos a}$	A1		
	$y_{max} = u\sin\theta\left(\dfrac{u\sin\theta}{g\cos a}\right) - \dfrac{1}{2}g\cos a\left(\dfrac{u\sin\theta}{g\cos a}\right)^2$	m1		
	$y_{max} = \dfrac{u^2\sin^2\theta}{2g\cos a}$	A1F	4	
	Total		**4**	

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General Certificate of Education

Mathematics 6360

MM03 Mechanics 3

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MM03

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1	$L = M^{\alpha}(LT^{-1})^{\beta}(LT^{-2})^{\gamma}$	M1A1		
	$\beta + \gamma = 1$ $-\beta - 2\gamma = 0$ $\alpha = 0$	m1		Getting three equations
	$\gamma = -1$	m1		Solution
	$\beta = 2$	A1F	5	
	Total		**5**	
2(a)	$x = 2t$	M1		
	$y = -\frac{1}{2}gt^2 + 10t$	M1		
	$t = \frac{x}{2}$			
	$y = -\frac{1}{2}g\left(\frac{x}{2}\right)^2 + 10\left(\frac{x}{2}\right)$	m1		
	$y = -\frac{g}{8}x^2 + 5x$	A1	4	AG
(b)	$1 = -\frac{g}{8}x^2 + 5x$	M1		
	$gx^2 - 40x + 8 = 0$			
	$x = \frac{40 \pm \sqrt{(-40)^2 - 4 \times 8g}}{2g}$	M1		
	$x = 3.871,\ 0.211$	A1		A1 for both answers
	$\text{Distance} = 3.66\,\text{m}$	A1	4	
(c)	$t = \frac{3.66}{2}$	M1		
	$t = 1.83$ sec	A1	2	
	Total		**10**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	${}_{\mathrm{P}}v_{\mathrm{F}} = \sqrt{4^2+2^2}$	M1		
	$= 4.47 \text{ m s}^{-1}$ or $2\sqrt{5}\ ms^{-1}$ or $\sqrt{20}\ ms^{-1}$	A1		
	$\theta = \tan^{-1}\dfrac{2}{4}$	M1		
	$\theta = 26.6°$	A1F		
	Bearing $= 40° + 180° - 26.6°$			
	$= 193°$	A1F	5	
	Alternative:			
	Comp. due west $= 4\sin 40° - 2\sin 50° = 1.04 \text{ m s}^{-1}$ Comp. due south $= 2\cos 50° + 4\cos 40° = 4.35 \text{ m s}^{-1}$	(M1)		OE; resolving in two directions
	${}_{\mathrm{P}}v_{\mathrm{F}} = \sqrt{1.04^2 + 4.35^2} = 4.47 \text{ m s}^{-1}$	(A1)		
	$\theta = \tan^{-1}\dfrac{1.04}{4.35}$ or $\tan^{-1}\dfrac{4.35}{1.04}$	(M1)		
	$\theta = 13.4°$ or $76.6°$	(A1F)		
	Bearing $= 13.4° + 180°$ or $270° - 76.6°$			
	$= 193°$	(A1F)		
	Alternative:			
	Correct triangle	(M1)		Any orientation
	${}_{P}v_{F} = \sqrt{1.04^2 + 4.35^2} = 4.47\text{ms}^{-1}$	(A1)		
	Rel. Vel. Triangle angle $26.6°$ or $63.4°$	(A1)		
	Bearing $= 40° + 180° - 26.6°$ or $63.4° + 40° + 90°$	(M1)		
	$= 193°$	(A1F)		
(b)(i)	$v_{\mathrm{F}} = v_{\mathrm{P}} + {}_{\mathrm{P}}v_{\mathrm{F}}$			
	$\dfrac{\sin\alpha}{2} = \dfrac{\sin 140°}{4}$	M1A1		
	$\alpha = 18.7°$	A1F		
	Bearing $= 90° + 18.7°$			
	$= 109°$	A1F	4	
	Alternative:			
	$2\sin 40° = 4\sin\alpha$	(M1)		
	$\alpha = \sin^{-1}\left(\dfrac{1}{2}\sin 40°\right)$	(A1)		
	$\alpha = 18.7°$	(A1F)		
	Bearing $= 109°$	(A1F)		

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(b)(ii)	$\beta = 180° - (140° + 18.7°)$ $= 21.3°$	B1F		
	$\dfrac{{}_{\text{P}}v_{\text{F}}}{\sin 21.3°} = \dfrac{4}{\sin 140°}$	M1		
	${}_{\text{P}}v_{\text{F}} = 2.2568\,\text{m s}^{-1}$	A1F		
	$t = \dfrac{1500}{2.2568}$ $= 665$ sec	A1F	4	
	Alternative:			o.e. resolving in two directions
	${}_{\text{F}}v_{\text{P}} = 4\cos 18.7 - 2\cos 40 = 2.2568$	(M1) (A2,1,0)		
	$t = \dfrac{1500}{2.2568} = 665$ sec	(A1F)		
(iii)	No cross wind, calm lake, instantaneous change of direction by the patrol boat	B1	1	Any sensible assumption
	Total		**14**	
4(a)	$\text{I} = \int_0^4 (t^3 + t)\,\text{d}t$	M1		
	$= \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^4$	m1		
	$= 72$ N s	A1	3	
(b)	$72 = 0.5v - 0.5(0)$	M1		Condone $-5(0)$
	$v = 144$	A1F	2	
(c)	$\int_0^T (t^3 + t)\,\text{d}t = 0.5(12) - 0.5(0)$	M1		Condone $-5(0)$
	$\left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^T = 6$			
	$T^4 + 2T^2 - 24 = 0$	A1		
	$T^2 = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-24)}}{2(1)}$	m1 A1F		
	or $(T^2 - 4)(T^2 + 6) = 0$			
	$T^2 = 4$ $T = 2$	A1F	5	
	Total		**10**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Momentum of B perpendicular to the line of centres is unchanged			
	$m_B v \sin 40° = 3m_B$	M1A1		
	$v = 4.667\ \text{m s}^{-1} = 4.67\ \text{ms}^{-1}$ (3sf)	A1	3	AG
(b)	$e = \dfrac{4.67\cos 40°}{5\cos 30°}$	M1A1		
	$e = 0.826$	A1F	3	
(c)	Impulse on A = change in momentum of A along the line of centres			
	$= 0.5 \times 5\cos 30° = 2.165$	M1A1		
	$= 2.17$ Ns	A1	3	AG
(d)	$2.165 = m_B(4.667)\cos 40°$	M1A1		
	$m_B = 0.6056 = 0.606$ kg (3sf)	A1F	3	Condone use of premature rounding giving 0.605kg or 0.607 kg
	Total		**12**	
6(a)	$5mu + 7mu = mv_A + 7mv_B$	M1A1		Allow consistent use of positive or negative sign for v_A.
	$12u = v_A + 7v_B$			
	$e = \dfrac{-v_A + v_B}{4u}$	M1		
	$-v_A + v_B = 4eu$			
	$8v_B = 12u + 4eu$	m1		
	$v_B = \dfrac{u}{2}(e+3)$	A1	5	AG
(b)	$v_A = \dfrac{u}{2}(e+3) - 4eu$	M1		
	$v_A = \dfrac{u}{2}(3-7e)$	A1F		
	$\dfrac{u}{2}(3-7e) < 0$	M1		
	$3 - 7e < 0$			
	$e > \dfrac{3}{7}$	A1	4	AG
(c)	$w_B = \dfrac{u}{4}(e+3)$	M1		
	$\dfrac{u}{2}(7e-3) < \dfrac{u}{4}(e+3)$	M1		
	$2(7e-3) < e+3$			
	$13e < 9$	m1		
	$e < \dfrac{9}{13}$	A1	4	AG
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$y = 10t\sin 40° - \frac{1}{2}gt^2\cos 30°$	M1A1		
	$y = 0 \Rightarrow t = \frac{20\sin 40°}{g\cos 30°}$	A1	3	AG
(b)	$\dot{x} = 10\cos 40° + g\sin 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{x} = 15.08 \text{ m s}^{-1}$	A1		
	$\dot{y} = 10\sin 40° - g\cos 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1		
	$\dot{y} = -6.427 \text{ m s}^{-1}$	A1	4	Allow 3 sf
(c)	$\dot{x}$ will be unchanged	B1		
	Rebound $\dot{y} = 6.427 \times 0.5 = 3.214$	M1		Allow using 3 sf
	Rebound speed $= \sqrt{15.08^2 + 3.214^2}$	m1		
	$= 15.4 \text{ m s}^{-1}$	A1F	4	
	Total		**11**	
	TOTAL		**75**	

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Mathematics MM03

Mechanics 3

Mark Scheme

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM03

Q	Solution	Marks	Total	Comments
1	LT^{-1}	B1		For dimensions of u
	$LT^{-1} = M^{\alpha}L^{\beta}T^{\gamma} \times L^3 \times ML^{-3} \times LT^{-2}$	M1 A1		M1 for equation with five components
	$1 = \beta + 1$ $-1 = \gamma - 2$			
	$0 = \alpha + 1$	m1		Forming and solving equations (PI)
	$\beta = 0,\ \alpha = \text{-}1,\ \gamma = 1$			
	The dimensions of C are $M^{-1}T$	A1F	5	
	Alternative :			
	LT^{-1}	(B1)		For dimensions of u
	$LT^{-1} = C \times L^3 \times ML^{-3} \times LT^{-2}$	(M1A1)		M1 for equation with five components
	$LT^{-1} = C \times L\,MT^{-2}$	(m1)		
	The dimensions of C are $M^{-1}T$	(A1F)	5	
	Total		**5**	
2(a)(i)	$x = 80\cos\theta.\,t$	B1		
	$t = \dfrac{x}{80\cos\theta}$	B1		
	$y = 80\sin\theta.t - \dfrac{1}{2}gt^2$	B1		
	$y = 80\sin\theta\dfrac{x}{80\cos\theta} - \dfrac{1}{2}g(\dfrac{x}{80\cos\theta})^2$	M1		
	$y = x\tan\theta - \dfrac{gx^2}{12800}(1+\tan^2\theta)$	A1	5	Answer given
(ii)	$-20 = 400\tan\theta - \dfrac{9.8\times 400^2}{12800}(1+\tan^2\theta)$	M1		Condone + 20
	$122.5\tan^2\theta - 400\tan\theta + 102.5 = 0$			
	$49\tan^2\theta - 160\tan\theta + 41 = 0$	A1	2	Answer given
(b)(i)	$tan\,\theta = \dfrac{160 \pm \sqrt{25600 - 4(49)(41)}}{2\times 49}$	M1		
	= 2.9850, 0.2803	A1		PI
	$\theta = 71.5^\circ,\ 15.7^\circ$	A1F	3	
(ii)	For the shortest time			
	400 = $80\cos 15.7^\circ.t$	M1		
	t =5.19	AIF	2	
(c)	• The projectile is a particle • The air resistance is negligible	E1	1	
	Total		**13**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
3(a)	C.L.M. $(1)3u=(1)v_A+(3)v_B$ Restitution: $\frac{1}{3}\times 3u=v_B-v_A$ $v_B=u$ $v_A=0$	M1 A1 M1 A1 m1 A1	6	M1 for three non-zero terms Accept v_A-v_B Solution A1 for both answers
(b)	C.L.M. $3u=3w_B+xw_c$ Restitution: $\frac{1}{3}u=w_C-w_B$ $w_C=\frac{4u}{3+x}$ $w_B=\frac{u(9-x)}{3(3+x)}$ OE	M1 A1 M1 A1 m1 A1	6	Solution attempt, dep. on both M1s AG A1 for both
(c)	For further collision $\frac{u(9-x)}{3(3+x)}<0$ $9u-xu<0$ $x>9$	M1 A1	2	AG
(d)	$I=5(\frac{4u}{3+5})$ $I=\frac{5u}{2}$ **Alternative:** $I=3u-3\times\frac{u(9-5)}{3(3+5)}$ $I=\frac{5u}{2}$	M1 A1 (M1) (A1F)	2	Accept $-\frac{5u}{2}$ Follow through on their w_B
	Total		**16**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$r_A = (-60\mathbf{i}+30\mathbf{k})+(250\mathbf{i}+50\mathbf{j}-100\mathbf{k})t$	M1		For correct form
	$r_B = (-40\mathbf{i}+10\mathbf{j}-10\mathbf{k})+(200\mathbf{i}+25\mathbf{j}+50\mathbf{k})t$	A1,2	3	A1 for each
(b)	${}_Br_A = [\,(-60\mathbf{i}+30\mathbf{k})+(250\mathbf{i}+50\mathbf{j}-100\mathbf{k})t\,]-$	M1		Attempt at the difference using their answers
	$[(-40\mathbf{i}+10\mathbf{j}-10\mathbf{k})+(200\mathbf{i}+25\mathbf{j}+50\mathbf{k})t]$			
	${}_Br_A = (-20+50t)\mathbf{i}+(-10+25t)\mathbf{j}+(40-150t)\mathbf{k}$	A1	2	AG
(c)	For collision			
	$(-20+50t)\mathbf{i}+(-10+25t)\mathbf{j}+(40-150t)\mathbf{k}=0$	M1		
	$-20+50t=0 \quad \Rightarrow \quad t=\frac{2}{5}$			
	$-10+25t=0 \quad \Rightarrow \quad t=\frac{2}{5}$	m1 A1F		
	$40-150t=0 \quad \Rightarrow \quad t=\frac{4}{15}$			
	The relative position vector cannot be zero.			
	Therefore A and B do not collide	E1	4	
(d)	$S^2 = (-20+50t)^2+(-10+25t)^2+(40-150t)^2$	M1A1		
	For minimum S			
	$\frac{dS^2}{dt} = 100(-20+50t)+50(-10+25t)-$	M1		
	$300(40-150t)=0$	A1F		
	$51250t-14500=0$	m1		Solution
	$t=0.283$	A1F	6	
	Total		**15**	
	Alternative:			
	$\begin{pmatrix} -20+50t \\ -10+25t \\ 40-150t \end{pmatrix} . \begin{pmatrix} 50 \\ 25 \\ -150 \end{pmatrix} = 0$	(M1) (A1)		
	$-1000+2500t-250+625t-6000+22500t=0$	(m1) (A1F)		
	$25625t-7250=0$	(A1F)		
	$t=0.283$	(A1F)		

MM03 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Parallel to the wall			
	$4\cos\alpha = v\cos 40^\circ$	M1		Correct trigonometric ratios
	Perpendicular to the wall			
	$v\sin 40^\circ = \frac{2}{3}\times 4\sin\alpha$	M1		Correct trigonometric ratios
	$\tan\alpha = \frac{3}{2}\tan 40^\circ$	A1	3	AG
(b)	$\alpha = 51.5^\circ$	M1		
	$v = \frac{4\cos 51.5^\circ}{\cos 40^\circ}$	M1		
	$v = 3.25 \quad \text{ms}^{-1}$	A1	3	OE
	Total		**6**	
6(a)	The spheres are smooth, no force acting in $\mathbf{j}$ direction	E1	1	Any valid reason
(b)	$v_A = a\mathbf{i} + b\mathbf{j}$			
	$v_B = c\mathbf{i} + d\mathbf{j}$			
	C.L.M. along **i**: $1(2) + 2(-1) = 1(a) + 2(c)$	M1A1		
	$a + 2c = 0$			
	Restitution along $\mathbf{i}$: $c - a = 0.5(2-(-1))$	M1A1		
	$c - a = 1.5$			
	$c = 0.5$			
	$a = -1$			
	$v_A = -\mathbf{i} + 3\mathbf{j}$	A1F		
	$v_B = 0.5\mathbf{i} - 2\mathbf{j}$	A1F	6	
	Total		**7**	

MM03 (cont)

Q	Solution	Marks	Total	Comments
7(a)	On striking *A*:			
	$20\sin 30^\circ.t - \frac{1}{2}(9.8)\cos 35^\circ.t^2 = 0$	M1A1		
	$t = 2.49$	A1		AWRT OE
	Components of Velocity :			
	$u_x = 20\cos 30^\circ - 9.8\sin 35^\circ(2.49)$	M1		
	$u_x = 3.32$	A1F		AWRT
	$u_y = 20\sin 30^\circ - 9.8\cos 35^\circ(2.49)$	M1		
	$u_y = -10$ (or -9.99)	A1F	7	
(b)	On Rebounding			
	$v_x = 3.32$			
	$v_y = \frac{4}{5} \times 10$	B1F		For $\frac{4}{5} \times$ their u_y
	$v_y = 8$ (or 7.99)			
	The rebound angle = $\tan^{-1}\frac{8}{3.32}$	M1		
	= 67.5° (or 67.4°)	A1F		
	$35^\circ + 67.5^\circ = 102.5^\circ$	M1 A1F		
	$102.5^\circ > 90^\circ$, therefore the second strike will be at a point lower down than *A*.	E1	6	Dependent on the two M1s
	Alternative:			
	$\frac{4}{5} \times 10 = 8$	(B1)		Condone negative sign
	$0 = 8t - \frac{1}{2}g\cos 35.t^2$	(M1)		
	$t = 1.9931$	(A1)		OE
	$x = 3.32t - \frac{1}{2}g\sin 35.t^2$	(M1)		
	$x = -4.55$ or -4.56	(A1)		
	The second strike will be at a point lower down than *A*.	(E1)		
	Total		**13**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2011

Mathematics MM03

(Specification 6360)

Mechanics 3

Final

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Further copies of this Mark Scheme are available from: aqa.org.uk

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MM03

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1 (a)	I = 0.2(32) + 0.2(18)	M1		
	I = 10 Ns	A1	2	Condone +10
(b)	$\int_0^{0.09} k(0.9\text{t-}10t^2)\mathrm{d}t = 10$	M1		Condone limits
	$k\left[0.45t^2 - \frac{10}{3}t^3\right]_0^{0.09} = 10$	A1F		Condone limits
	$1.215\times10^{-3}k = 10$	m1		For substituting 0.09
	$k = 8230$	A1F	4	
			6	
2	$\mathrm{T}^1 = \mathrm{L}^{\alpha}(\mathrm{MLT}^{-2})^{\beta}(\mathrm{ML}^{-1})^{\gamma}$	M1 A1		
	$\alpha + \beta - \gamma = 0$ $\beta + \gamma = 0$ $-2\beta = 1$	m1		Getting three equations
	$\beta = -\frac{1}{2}$	m1		
	$\gamma = \frac{1}{2}$ $\alpha = 1$	A1F	5	Solution
			5	

Q	Solution	Marks	Total	Comments
3 (a)	$x = 40\cos\theta . t$	M1		
	$y = -\frac{1}{2}(10)t^2 + 40\sin\theta . t$	M1 A1		
	$y = -\frac{1}{2}(10)(\frac{x}{40\cos\theta})^2 + 40\sin\theta .(\frac{x}{40\cos\theta})$	m1		Dependent on both M1s
	$y = -\frac{x^2}{320\cos^2\theta} + x\tan\theta$			
	$320y = -x^2(1+\tan^2\theta) + 320x\tan\theta$	m1		
	$x^2\tan^2\theta - 320x\tan\theta + (x^2 + 320y) = 0$	A1	6	Answer Given (Condone missing brackets)
(b)(i)	$150^2\tan^2\theta - 320(150)\tan\theta + (150^2 + 320\times 8) = 0$	M1		
	$1125\tan^2\theta - 2400\tan\theta + 1253 = 0$	A1		Correct quadratic
	$\tan\theta = \frac{2400 \pm \sqrt{2400^2 - 4(1125)(1253)}}{2(1125)}$	m1		
	$\tan\theta = 1.22\ ,\ 0.912$	A1F		PI
	$\theta\ =\ 50.7^\circ\ ,\ 42.4^\circ$	A1F	5	
(b)(ii)	$\theta = 42.4^\circ$	B1F		For the smaller angle
	$t = \frac{150}{40\cos\theta}$ and $\cos 42.4 > \cos 50.7$	E1	2	OE
			13	

Q	Solution	Marks	Total	Comments
4 (a)	$u_A = \dfrac{(-2\mathbf{i}+3\mathbf{j}+6\mathbf{k})140}{\sqrt{(2)^2+(3)^2+(6)^2}} = -40\mathbf{i}+60\mathbf{j}+120\mathbf{k}$	M1 A1		Simplification not needed
	$u_B = \dfrac{(2\mathbf{i}-\mathbf{j}+2\mathbf{k})60}{\sqrt{(2)^2+(1)^2+(2)^2}} = 40\mathbf{i}-20\mathbf{j}+40\mathbf{k}$	A1		Simplification not needed
	${}_Au_B = (-40\mathbf{i}+60\mathbf{j}+120\mathbf{k})-(40\mathbf{i}-20\mathbf{j}+40\mathbf{k})$ $= -80\mathbf{i}+80\mathbf{j}+80\mathbf{k}$	M1 A1F	5	Subtracting *B* from *A*
(b)	${}_Ar_B = (4\mathbf{i}-2\mathbf{j}+3\mathbf{k})-(-3\mathbf{i}+6\mathbf{j}+3\mathbf{k})+ t(-80\mathbf{i}+80\mathbf{j}+80k)$ **or** $(7\mathbf{i}-8\mathbf{j})+t(-80\mathbf{i}+80\mathbf{j}+80\mathbf{k})$	M1 A1F	2	A difference of initial p.v. $+t\times {}_Au_B$
(c)	${}_Ar_B = (7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}$	B1F		
	$s^2 = (7-80t)^2+(-8+80t)^2+(80t)^2$	B1F		
	$2s\dfrac{\mathrm{d}s}{\mathrm{d}t} = 2(7-80t)(-80)+2(-8+80t)(80)+ 2(80t)(80)=0$	M1 A1F		Differentiation
	$240t = 15$	m1		Solving
	$t = 0.0625$ or $\dfrac{1}{16}$	A1F		
	$s^2 = (7-80\times0.0625)^2+(-8+80\times0.0625)^2+ (80\times0.0625)^2$	M1		
	$s = 6.16$ km or $\sqrt{38}$ km	A1F	8	
			15	
	<u>Alternative</u> (Not in the specification) *A* and *B* are closest $\Rightarrow\ {}_A\mathrm{r}_B\ .\ {}_A\mathrm{v}_B = 0$ $[(7-80t)\mathbf{i}+(-8+80t)\mathbf{j}+(80t)\mathbf{k}]\ .$ $[-80\mathbf{i}+80\mathbf{j}+80\mathbf{k}]=0$ $-80(7-80t)+80(-8+80t)+80(80t)=0$ $240t=15$ $t=0.0625$	B1 M1 A1 A1 M1 A1		

Q	Solution	Marks	Total	Comments
5(a)	$v^2 = u^2 + 2as$ $v^2 = 0^2 + 2(9.8)(2.5)$ $v = 7$	 M1 A1	 2	
(b)(i)	$\frac{w}{7} = e$ $w = 7e$ $0 = 7et - \frac{9.8}{2}t^2$ **or** **(** $0 = 7e - 9.8t$ **)** $t = \frac{10e}{7}$ **(** $t = 2\times\frac{7e}{9.8}$ **)**	M1 M1 A1	 3	 Answer given
(ii)	$w' = 7e^2$ $0 = 7e^2t' - \frac{9.8}{2}t'^2$ $t' = \frac{10e^2}{7}$	 B1	 1	 OE
(c)	$0^2 = (7e)^2 + 2(-9.8)h_2$ $h_2 = 2.5e^2$ $h_3 = 2.5e^2$ $0^2 = (7e^2)^2 + 2(-9.8)h_4$ $h_4 = 2.5e^4$ $h_5 = 2.5e^4$ Total distance = $2.5 + 2(2.5e^2) + 2(2.5e^4)$ $= 2.5 + 5e^2 + 5e^4$	M1 A1 A1 m1 A1	 5	Or for correct method to find h_4
	Alternative (not in the specification) K.E. after each bounce $= e^2 \times$ K.E. before the bounce P.E. at max. height after each bounce $=$ $e^2 \times$ P.E. at max. height before the bounce Height after first bounce = $2.5e^2$ Height after second bounce $= 2.5e^4$ Total = $2.5 + 2(2.5e^2 + 2(2.5e^4)$ = $2.5 + 5e^2 + 5e^4$	 (M1) (A1) (A1) (m1) (A1)		
(d)	Motion in vertical line, No air resistance, No energy loss, Instantaneous bounce	B1	1	
			12	

Q	Solution	Marks	Total	Comments
6 (a)	Perpendicular to the plane:			
	$y=-\frac{1}{2}gt^2\cos 20+ut\sin 30$	M1		
	$0=-4.9t^2\cos 20+ut\sin 30$	M1		
	$t=0.108589568u$ or $\frac{2u\sin 30}{g\cos 20}$	A1		
	Parallel to the plane:			
	$x=-\frac{1}{2}gt^2\sin 20+ut\cos 30$	M1		
	$200=-4.9(0.108589568u)^2\sin 20+$ $u(0.108589568u)\cos 30$	m1		
	$u^2=2693$	A1F		
	$u=51.9$ or 51.894	A1F	7	Do not accept $\sqrt{2693}$
(b)	$\dot{y}=-gt\cos 20+u\sin 30=0$	M1		
	$t=2.817899$ or 2.817580214 or $\frac{51.9\sin 30}{g\cos 20}$	A1F		Accept 3 significant fig.
	The greatest $\perp$ distance $=$			
	$-\frac{1}{2}9.8(2.817899)^2\cos 20+51.9(2.817899)\sin 30$ or	m1		
	$-\frac{1}{2}9.8(\frac{51.894\sin 30}{9.8\cos 20})^2\cos 20+51.9(\frac{51.894\sin 30}{9.8\cos 20})\sin 30$			
	$=36.5622$ m or 36.5538 $=36.6$ 3sf	A1F	4	
			11	
6 (a)	**Alternative:**			
	$x=200\cos 20$	B1		
	$y=200\sin 30$	B1		
	$200\cos 20=u\cos 50t$	M1		
	$t=\frac{292.4}{u}$	A1		
	$200\sin 30=\frac{1}{2}(-9.8)(\frac{292.4}{u})^2+u\sin 50(\frac{292.4}{u})$	M1		
	$u^2=2693$	A1		
	$u=51.9$	A1		
(b)	**Alternative:**			
	$0=(u\sin 30)^2-2g\cos 20.s$	M1		
	$s=\frac{(51.9\sin 30)^2}{2(9.8)\cos 20}$	m1A1		
	$s=36.6$	A1		

Q	Solution	Marks	Total	Comments
7 (a)	Momentum of *A* is unchanged $\perp$ to the line of centres			
	$4mu\sin 30 = 4mv_A \sin\alpha$	M1		
	$v_A = \dfrac{u}{2\sin\alpha}$ ....(1)	A1		
	C.L.M.:			
	$4mu\cos 30 = 4mv_A\cos\alpha + 3mv_B$	M1A1		
	$2\sqrt{3}u = 4v_A\cos\alpha + 3v_B$ ....(2)	A1F		OE
	Restitution along the line of centres:			
	$\dfrac{v_B - v_A\cos\alpha}{u\cos 30} = \dfrac{5}{9}$	M1A1		
	$v_B = v_A\cos\alpha + \dfrac{5\sqrt{3}u}{18}$ ....(3)	B1		Or equivalent, could be in part (b)
	$2\sqrt{3}u = 4\dfrac{u}{2\sin\alpha}\cos\alpha + 3\dfrac{u}{2\sin\alpha}\cos\alpha + \dfrac{15\sqrt{3}u}{18}$	m1		Solving (1), (2) and (3) Dependent on three M1s
	$\dfrac{7\sqrt{3}}{6} = \dfrac{7}{2\tan\alpha}$			
	$\tan\alpha = \sqrt{3}$			
	$\alpha = 60^\circ$ or $\dfrac{\pi}{3}$	A1F	10	
(b)	Impulse on *B* = Change in momentum of *B* along the line of centres			
	$v_B = \dfrac{u}{2\sin 60}\cos 60 + \dfrac{5\sqrt{3}u}{18}$			
	$v_B = \dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}$ $\quad (= \dfrac{4\sqrt{3}}{9})$	M1		
	$\mathrm{I} = 3m(\dfrac{u}{2\sqrt{3}} + \dfrac{5\sqrt{3}u}{18}) - 3m(0)$	M1		
	$\mathrm{I} = \dfrac{4mu}{\sqrt{3}}$ or $2.31mu$	A1F	3	
			13	
	TOTAL		**75**	

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June 2012

Mathematics MM03

(Specification 6360)

Mechanics 3

Mark Scheme

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It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

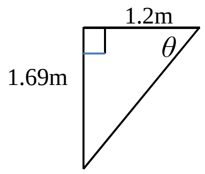
MM03

Q	Solution	Marks	Total	Comments
1(a)	$I = \int_0^{0.5} 4\times10^4 t^2(1-2t)\,\mathrm{d}t$	M1 A1		Attempt to integrate Use of correct limits, PI
	$= 4\times10^4\left[\frac{1}{3}t^3 - \frac{1}{2}t^4\right]_0^{0.5}$	A1F		Correct integration
	$= 417$ (or $\frac{1250}{3}$) Ns	A1F	4	Accept $416.\dot{6}$ or 416.7
(b)	$416.\dot{6} = 60v + 60\times5$	M1A1F		A1F correct sign
	$v = 1.94$	A1F	3	AWRT 1.94, accept 1.95 ISW
	Total		**7**	
2	Dimension of g is LT^{-2} Dimension of s is L Dimension of h is L Dimension of m_1 and m_2 is M	B1		B1 for dimensions of the five quantities
	Dimension of $\frac{g}{s}[s(m_1+m_2)+\frac{hm_1^2}{m_1+m_2}]$ is			
	$\frac{\mathrm{LT}^{-2}}{\mathrm{L}}[\mathrm{LM}+\frac{\mathrm{LM}^2}{\mathrm{M}}] \cong \mathrm{MLT}^{-2}+\mathrm{MLT}^{-2}$	M1		Correct substitution of dimensions
	$\cong \mathrm{MLT}^{-2}$	A1		
	which is a force	B1	4	
	Total		**4**	

MM03

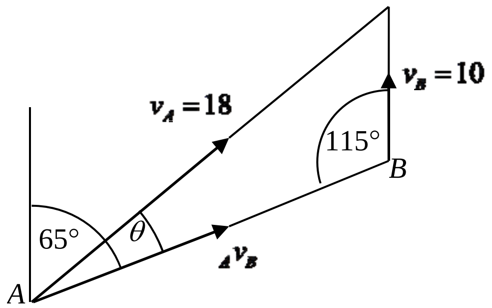
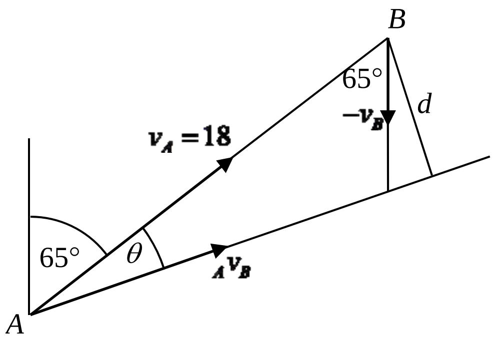
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\alpha$	M1		
	$t = \dfrac{x}{u\cos\alpha}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\alpha$	M1		Must have correct signs
	$y = -\dfrac{1}{2}g(\dfrac{x}{u\cos\alpha})^2 + u(\dfrac{x}{u\cos\alpha})\sin\alpha$	M1		
	$y = -\dfrac{gx^2}{2u^2\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\alpha) + x\tan\alpha$	A1		
	$k = -\dfrac{10(2k)^2}{2u^2}(1+\tan^2\alpha) + 2k\tan\alpha$	M1		
	$u^2 = -20k(1+\tan^2\alpha) + 2u^2\tan\alpha$			
	$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$	A1	7	AG
(b)	Pass through $P \Longrightarrow$ Discriminant ≥ 0			
	$(-2u^2)^2 - 4(20k)(u^2 + 20k) \geq 0$	M1A1		OE must be seen
	$4u^4 - 80ku^2 - 1600k^2 \geq 0$			
	$u^4 - 20ku^2 - 400k^2 \geq 0$	A1	3	AG
	Total		**10**	

MM03

Q	Solution	Marks	Total	Comments
4(a)				
	$\theta = \tan^{-1}\frac{1.69}{1.2} = 54.623^\circ$	B1		AWRT 55°
	$u\cos 60^\circ = v\cos 54.623^\circ$	M1		$v = 0.864u$
	$eu\sin 60^\circ = v\sin 54.623^\circ$	M1		
	$e = \dfrac{v\sin 54.623^\circ}{\dfrac{v\cos 54.623^\circ}{\cos 60^\circ}\times\sin 60^\circ}$	m1		OE, dependent on both M1s
	$e = 0.813 \quad \text{or} \quad 0.812$	A1	5	ISW
(b)	$I = 0.15u\sin 60^\circ + 0.15v\sin 54.623^\circ$	M1A1		Single angle values needed for A1
	$= \quad 0.15u\sin 60^\circ + 0.15\times\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}\times\sin 54.623^\circ$	m1		
	$= 0.236u$	A1	4	AG (condone 0.2355 or negative result)
(c)	Attempt at considering motion parallel or perpendicular to *AC*	M1		
	$t = \dfrac{1.2}{u\cos 60^\circ}$	M1		
	$t = \dfrac{12}{5u} \quad \text{or} \quad \dfrac{2.4}{u}$	A1	3	OE, No ISW
	Alternative:			
	$CP = \dfrac{1.2}{\cos 54.623^\circ} \qquad (= 2.072703844 \text{ m})$	(M1)		
	$t = \dfrac{\dfrac{1.2}{\cos 54.623^\circ}}{\dfrac{u\cos 60^\circ}{\cos 54.623^\circ}}$	(M1)		
	$= \dfrac{12}{5u} \quad \text{or} \quad \dfrac{2.4}{u}$	(A1)	(3)	(OE), No ISW
(d)	Velocity (momentum) parallel to the cushion is unchanged, or, Restitution only affects motion perpendicular to the cushion	E1	1	Accept 'horizontal component of velocity is unchanged'
	Total		**13**	

Q	Solution	Marks	Total	Comments
5(a)	$0 = 15t\sin 30 - \frac{1}{2}g\cos 25t^2$	M1A1		Accept wrong angle(s) for M1 but **not** sin and cos in wrong places
	$t = \dfrac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$t = 1.69$ sec.	A1F	4	AWRT 1.69
(b)	$\perp$ to plane $\dot{y} = 15\sin 30 - g\cos 25 \times \dfrac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{y} = -7.5$ ms^{-1}	A1F		Or -7.51, ft from their answer in (a)
	$\parallel$ to plane $\dot{x} = 15\cos 30 - g\sin 25 \times \dfrac{15\sin 30}{\frac{1}{2}g\cos 25}$	M1		
	$\dot{x} = 5.995766$ or 6.00 ms^{-1}	A1F		Accept 5.99
	Restitution: Rebound $\dot{y} = \frac{2}{3} \times 7.5 = 5$ ms^{-1}	M1		Or 5.01
	$\dot{x}$ unchanged	B1		PI, dependent on the last M1
	Speed of rebound $= \sqrt{5.995766^2 + 5^2}$ $= 7.81$ ms^{-1}	m1 A1F	8	Dependent on the last three M1s
	Total		**12**	

MM03

Q	Solution	Marks	Total	Comments
6(a)		B1		For any appropriate diagram PI by correct method
	$\frac{\sin\theta}{10} = \frac{\sin 115^\circ}{18}$	M1		
	$\theta = 30.2^\circ$	A1		
	Bearing $= 035^\circ$	A1	4	Accept 034.8°
(b)(i)		B1		For any appropriate diagram PI by correct method
	${}_Av_B{}^2 = 18^2 + 10^2 - 2(18)(10)\cos 65^\circ$	M1		
	${}_Av_B = 16.4881 \quad \text{ms}^{-1}$	A1		OE
	$\frac{\sin 65^\circ}{16.4881} = \frac{\sin\theta}{10}$	M1		
	$\theta = 33.3446^\circ$	A1F		
	$d = 12 \times \sin 33.3446^\circ$	m1		OE
	$d = 6.60 \quad \text{km}$	A1F	7	Dependent on the previous two M1s (AWRT 6.6 km)
(ii)	$t = \frac{12 \times \cos 33.3446^\circ}{16.4881} = 0.607987$ hours $(= 36.5 \quad \text{min})$	M1 A1F A1F	3	Or 0.608 hours LHS values Correct time
	Total		**14**	

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Q6 (b)(i) Alternative:

$r_A = [(18\cos 25)\mathbf{i} + (18\sin 25)\mathbf{j}]t$	
$r_B = [(12\cos 25)\mathbf{i} + (12\sin 25)\mathbf{j}] + 10\mathbf{j}t$	M1 for both
${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$	m1 A1
$\lvert {}_A r_B \rvert^2 = (-12\cos 25 + 18t\cos 25)^2 + (-12\sin 25 + 18t\sin 25 - 10t)^2$	A1
$\frac{\mathrm{d}\lvert {}_A r_B \rvert^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$	m1
$t = 0.608$	A1
$d = 6.60$ km or 6.6 km	A1
The corresponding marks awarded for finding the closest approach time:	
$\frac{\mathrm{d}\lvert {}_A r_B \rvert^2}{\mathrm{d}t} = (36\cos 25)(-12\cos 25 + 18t\cos 25) + (36\sin 25 - 20)(-12\sin 25 + 18t\sin 25 - 10t) = 0$	M1 A1
$t = 0.608$ (or better)	A1

(b)(i) Alternative (**Not in the specification**):

${}_A r_B = (-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}$	M1 A1
$[(-12\cos 25 + 18t\cos 25)\mathbf{i} + (-12\sin 25 + 18t\sin 25 - 10t)\mathbf{j}] \cdot [(18\sin 65)\mathbf{i} + (18\cos 65 - 10)\mathbf{j}] = 0$	m1
$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$	A1
$271.85\,t = 165.27$	m1
$t = 0.608$ (or better)	A1
$d = 6.60$ km or 6.6 km	A1
The corresponding marks awarded for finding the closest approach time:	
$(-12\cos 25 + 18t\cos 25)(18\sin 65) + (-12\sin 25 + 18t\sin 25 - 10t)(18\cos 65 - 10) = 0$	M1
$271.85\,t = 165.27$	A1
$t = 0.608$ (or better)	A1

(b)(ii) FT from their answers in part (b)(i)

MM03

Q	Solution	Marks	Total	Comments
7(a)	$2m(3i+j)+m(2i-5j)=2mv_A+m(2i+j)$	M1A1		
	$8i-3j=2v_A+(2i+j)$			
	$v_A=3i-2j$	A1	3	
(b)	$I=m(2i+j)-m(2i-5j)$	M1A1		
	$I=6mj$	A1	3	AG
(c)	$\mathrm{I}=6mj \quad \Rightarrow$ Line of centres along j	B1		PI
	Restitution along j: $\quad 1+2=e(5+1)$	M1A1		
	$e=0.5$	A1	4	
	Accept energy methods			
(d)	${}_Av_B=i-3j$			
	${}_Ar_B=-0.1j+(i-3j)t$	M1A1		
	$1.1^2=t^2+(-0.1-3t)^2$	M1		OE
	$10t^2+0.6t-1.2=0$			
	$t=\dfrac{-0.6\pm\sqrt{0.6^2-4(10)(-1.2)}}{2(10)} \qquad (=0.31770677)$	m1		Dependent on both M1s
	$t=0.318$ or 0.317 sec.	A1	5	
	Total		**15**	
	TOTAL		**75**	

Version 1.0

General Certificate of Education (A-level)
June 2013

Mathematics **MM03**

(Specification 6360)

Mechanics 3

Final

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Otherwise we require evidence of a correct method for any marks to be awarded.

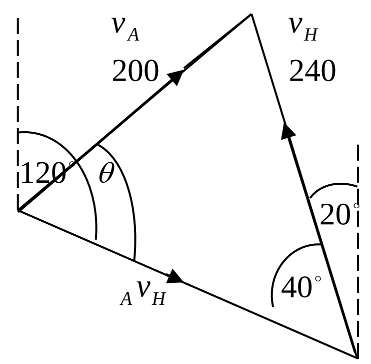
Q	Solution	Marks	Total	Comments
1	Use of Impulse-momentum principle	M1		$\int_{(0)}^{(T)} (3t+1)\,\mathrm{d}t = \pm 2(5) \pm 2(1)$ Condone sign error for M1
	$\int_{(0)}^{(T)} (3t+1)\,\mathrm{d}t = 2(5) - 2(1)$	A1		A1 for all correct
	$\left[\frac{3}{2}t^2 + t\right]_{(0)}^{(T)} = (8)$	A1		Correct integration, PI by the correct quadratic
	$3T^2 + 2T - 16 = 0$	A1		Correct use of correct limits and rearrangement
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{4 - 4(3)(-16)}}{2(3)}$	m1		Solution of their quadratic, **correct** attempt needed
	($T = -\frac{8}{3}$ unacceptable, not in the interval $0 \le t \le 3$) $\underline{T = 2}$ $\sqrt{\quad}$	A1	6	
	Total		**6**	
2	$[P]$ = $\mathrm{MLT^{-2}.\,L\,.\,T^{-1}} = \mathrm{ML^2T^{-3}}$	B1		For correct unsimplified dimensions of quantities
	$[mgv\sin\theta]$ = $\mathrm{M\,.\,LT^{-2}.\,LT^{-1}} = \mathrm{ML^2T^{-3}}$	B1		
	$[Rv]$ = $\mathrm{MLT^{-2}.\,LT^{-1}} = \mathrm{ML^2T^{-3}}$	B1		
	$\left[\frac{1}{2}mv^3\frac{\sin\theta}{h}\right]$ = $\mathrm{M\,.\,L^3T^{-3}.\,L^{-1}} = \mathrm{ML^2T^{-3}}$	B1		
		B1		All simplifications correct
	The formula is dimensionally consistent	E1		Dependent on the last B1
	Total		**6**	

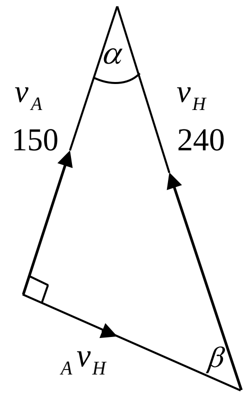
Q	Solution	Marks	Total	Comments
3(a)	$x = ut\cos\theta$	M1		
	$t = \dfrac{x}{u\cos\theta}$	A1		
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	M1		Condone $+g$ for M1
	$y = -\dfrac{1}{2}gt^2 + ut\sin\theta$	A1		
	$y = -\dfrac{1}{2}g(\dfrac{x}{u\cos\theta})^2 + u(\dfrac{x}{u\cos\theta})\sin\theta$	m1		Elimination of *t*, condone $+ g$ for m1
	$y = -\dfrac{gx^2}{2u^2\cos^2\theta} + \dfrac{x\sin\theta}{\cos\theta}$			
	$y = -\dfrac{gx^2}{2u^2}(1+\tan^2\theta) + x\tan\theta$	A1	6	OE in terms of x, u, g, $\tan\theta$
(b)(i)	$0.5 = -\dfrac{9.8(5)^2}{2(8)^2}(1+\tan^2\theta) + 5\tan\theta$	M1		Correctly substituting for x, y, u and g into their equation of trajectory
		A1		All correct, condone decimal approximation.
	$245\tan^2\theta - 640\tan\theta + 309 = 0$	A1		OE exact quadratic in $\tan\theta$
	$\tan\theta = \dfrac{640 \pm \sqrt{(-640)^2 - 4(245)(309)}}{2(245)}$	m1		PI by the values of $\tan\theta$
	$\tan\theta = 1.973(004)$, $0.6392(41)$			
	$\theta = 63.12^\circ$, 32.58°			
	$\theta = 63.1^\circ$, 32.6°	A1	5	AG Must see the above or more accurate values
(ii)	$\dot{y} = -9.8(\dfrac{5}{8\cos 63.1^\circ}) + 8\sin 63.1^\circ$ OE	M1		Condone +9.8 for M1.
	$(\dot{y} = -6.4035)$			
	$\dot{x} = 8\cos 63.1^\circ$	M1		
	$(\dot{x} = 3.6195)$			
	$\tan^{-1}\dfrac{6.4(035)}{3.6(195)}$ $(= 61^\circ)$ OE	m1		PI by correct angle in a statement
	Direction : 61° to the horizontal or 29° to the vertical	A1	4	Have to see “horizontal” or “vertical” or diagram

Q	Solution	Marks	Total	Comments
3(b)(ii)	**Alternative:**			
	$\frac{dy}{dx} = -\frac{2gx}{2u^2}(1+\tan^2\theta)+\tan\theta$	(M1)		
	$= -\frac{2\times 9.8\times 5}{2\times 8^2}(1+\tan^2 63.1^\circ)+\tan 63.1^\circ$ $=-1.7692$	(A1)		
	$\tan^{-1}(-1.7692) = -60.52368^\circ$	(m1)		
	Direction: 61° to the horizontal or 29° to the vertical	(A1)		
(c)	The ball is a particle, or No air resistance, or The ball does not spin	B1	1	
	Total		**16**	
4(a)	$m(4u)+3m(2u) = mv_A + 3mv_B$	M1 A1		M1 for four correct momentum terms with any signs. A1for all correct
	$\frac{v_B - v_A}{4u-2u} = e$	M1 A1		M1 for correct terms for any signs, A1 for all correct.
	$\begin{pmatrix} v_A + 3v_B = 10u \\ v_B - v_A = 2ue \\ 4v_B = 2ue + 10u \end{pmatrix}$			
	$v_B = \frac{u}{2}(e+5)$	A1		OE, simplified
	$\left(v_A = \frac{u}{2}(e+5) - 2ue\right)$			
	$v_A = \frac{u}{2}(-3e+5)$	A1	6	OE, simplified
(b)	$e \le 1 \Rightarrow v_B \le \frac{u}{2}(1+5)$	M1		Use of $e \le 1$ (OE) needed FT their v_B
	$\Rightarrow v_B \le 3u$	A1	2	
(c)	$(I =)\ 3m.\frac{u}{2}(\frac{2}{3}+5) - 3m.2u$	M1		M1 for a difference of two momentums FT their velocity from part (a)
		A1F		A1F for their '*Final B – Initial B*'
	$= \frac{5mu}{2}$ or $2.5mu$	A1	3	
	Total		**11**	

Q	Solution	Marks	Total	Comments
5(a)	$\perp$ to plane $y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	M1		For M1, $\sin\alpha$ and $\cos\theta$ must be in the correct terms but accept $+g$.
	$y = ut\sin\alpha - \frac{1}{2}gt^2\cos\theta$	A1		
	$uT\sin\alpha - \frac{1}{2}gT^2\cos\theta = 0$	m1		Accept $+g$ for m1.
	$u = \dfrac{Tg\cos\theta}{2\sin\alpha}$	A1	4	OE
(b)	t or $T = \dfrac{2u\sin\alpha}{g\cos\theta}$	B1		
	$\parallel$ to plane $x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	M1		For M1, $\cos\alpha$ and $\sin\theta$ must be in the correct terms but accept $-g$.
	$x = ut\cos\alpha + \frac{1}{2}gt^2\sin\theta$	A1		
	$(\overline{OP} =)\ u(\dfrac{2u\sin\alpha}{g\cos\theta})\cos\alpha + \frac{1}{2}g(\dfrac{2u\sin\alpha}{g\cos\theta})^2\sin\theta$	m1		Elimination of t substituting their expression into their equation for x.
	$\left(= \dfrac{2u^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \dfrac{2u^2\sin^2\alpha\sin\theta}{g\cos^2\theta}\right)$			
	$= \dfrac{2u^2\sin\alpha(\cos\alpha\cos\theta + \sin\alpha\sin\theta)}{g\cos^2\theta}$	m1		OE single correct fraction in factorised form
	$= \dfrac{2u^2\sin\alpha\cos(\alpha-\theta)}{g\cos^2\theta}$	A1	6	AG Sight of the above line needed
	Total		**10**	

Q	Solution	Marks	Total	Comments
6	(Let $v_B = a\mathbf{i} - b\mathbf{j}$)			
	$\frac{a}{b} = \frac{3}{2}$	M1		Allow sign error
	$\frac{a}{b} = \frac{3}{2}$	A1		OE
	(Squares are smooth $\Rightarrow$ j component $\Rightarrow$) $b = 3$	B1		
	$a = \frac{9}{2}$	A1	4	AG
	$\left(v_B = \frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)$			
(b)	(C.L.M. along the line of centres:)			
	$4(4) - 2(2) = 4(v_A) + 2(\frac{9}{2})$	M1		OE, No sign errors
	$v_A = \frac{3}{4}$	A1		
	(Restitution along the line of centres:)			
	$e = \dfrac{-\frac{3}{4} + \frac{9}{2}}{4+2}$ OE	M1 A1		M1 for correct terms, A0 for sign error
	$e = \frac{5}{8}$	A1	5	
(c)	(I = Change in momentum of B along the line of centres)			
	$= 2\,(\frac{9}{2}\mathbf{i}) - 2\,(-2\mathbf{i})$	M1		Allow sign error and missing **i**
	$= 13\mathbf{i}$	A1		A0 for magnitude or $-13\mathbf{i}$
	Ns or kg m s^{-1}	B1	3	
	Total		**12**	

Q	Solution	Marks	Total	Comments
7(a)(i)		B1 B1		Correct diagram with or without arrows. 40° marked correctly, PI by correct method.
	$\frac{\sin\theta}{240} = \frac{\sin 40}{200}$	M1		Correct sine rule allowing their angle opposite 200 in their diagram.
	$\theta = 50.47483^\circ$ or 50.5°	A1		AWRT 50.5°, PI by correct bearing
	Bearing of $v_A = 069.5^\circ$	A1	5	Allow 69.5°
(a)(ii)	$\frac{_Av_H}{\sin(180^\circ - 40^\circ - 50.5^\circ)} =$ $\frac{200}{\sin 40^\circ}$ or $\frac{240}{\sin 50.5^\circ}$	M1		Allow using their angle from part (a)(i).
	$_Av_H = 311.13408$ or 311	A1F		FT their angle from part (a)(i)
	Time $= \frac{20}{311.13408}$	M1		PI by correct answer. Allow their $_Av_H$.
	$(= 0.0642809 \text{ hours}) = 3.86$ min	A1F	4	3sf required

Q	Solution	Marks	Total	Comments
7(b)		M1 A1		Right-angled triangle with 240 and 150 marked. Correct orientation
	$\cos\alpha = \frac{150}{240}$ or $\sin\beta = \frac{150}{240}$	M1		
	$\alpha = 51.3^\circ$ or $\beta = 38.7^\circ$	A1		PI by correct bearing
	Bearing: 031.3°	A1	5	Allow 31.3°
	Total		**14**	
	TOTAL		**75**	

A-LEVEL

Mathematics

Mechanics 3 – MM03

Mark scheme

6360

June 2014

Version/Stage: 1.0 Final

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M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

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Q	Solution	Mark	Total	Comment
1 (a)	$x = 4\sqrt{3}t$	B1		
	$y = 4t - \frac{1}{2}gt^2$	B1		
	$t = \frac{x}{4\sqrt{3}}$			
	$y = 4 \times \frac{x}{4\sqrt{3}} - \frac{1}{2}(9.8)\left(\frac{x}{4\sqrt{3}}\right)^2$	M1		
	$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$	A1	4	AG
(b)	$y = \frac{4}{\sqrt{3}} - \frac{49(4)^2}{480}$	M1		PI by correct answer
	(The height is $0.676 + 0.3$) 0.98 m or 98 cm	A1	2	CAO
(c)	No air resistance or The ball does not spin **or** No loss of energy	B1	1	
	Total		**7**	

Q	Solution	Mark	Total	Comment
2	$\left.\begin{array}{l}[J] \equiv \text{MLT}^{-1} \\ [g] \equiv \text{LT}^{-2}\end{array}\right\}$	B1		Dimensions of J and g, PI
	$\text{MLT}^{-1} = L^{\alpha}\left(\text{ML}^2\right)^{\beta}\left(\text{LT}^{-2}\right)^{\gamma}$	M1		FT from B1
	$\text{MLT}^{-1} = \text{M}^{\beta}\text{L}^{\alpha+2\beta+\gamma}\text{T}^{-2\gamma}$	A1		PI
	$\beta = 1$	B1		
	$-2\gamma = -1$ $\alpha + 2\beta + \gamma = 1$	m1		Correctly solving **their** two equations **involving three unknowns,** PI by the answers
	$\left.\begin{array}{l}\gamma = \frac{1}{2} \\ \alpha = -\frac{3}{2}\end{array}\right\}$	A1		
	Total		**6**	

(a)	Only quoting the formula and substituting scores M1 A1.

Q	Solution	Mark	Total	Comment
3 (a)	$I = \int_0^3 (3t+1)\,dt$	M1		Condone missing limits and missing dt
	$= \left[\frac{3}{2}t^2 + t\right]_0^3$	m1		For correct integration only
	$= \frac{33}{2}$ or 16.5 Ns	A1	3	Condone missing units
(b)	$\frac{33}{2} = 0.5v - 0.5(4)$	M1		Impulse/momentum equation for correct terms, FT on their impulse from part (a)
	$v = 37$ ms^{-1}	A1F	2	
(c)	$\int_0^T (3t+1)\,dt = 0.5(20) - 0.5(4)$	M1		Correct impulse-momentum equation, condone missing limits
	$\left[\frac{3}{2}t^2 + t\right]_0^T = 0.5(20) - 0.5(4)$			
	$3T^2 + 2T - 16 = 0$	A1		Correct quadratic equation
	$(3T+8)(T-2) = 0$ or $T = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-16)}}{2(3)}$	m1		Correct solution of their equation, PI
	$T = 2$ s	A1		Rejecting impossible time PI
	$\left(T = -\frac{8}{3} \text{ s impossible}\right)$		4	
	Total		**9**	

(a)

Alternative (non-calculus): Attempt at finding the area under force-time graph M1

$= \frac{1+10}{2} \times 3$ OE A1

$= 33/2$ or 16.5 (NS) A1

(c)

Alternative:

$a = \frac{3t+1}{0.5}$

$v = \int \frac{3t+1}{0.5}(dt)$ Attempt at integrating the acceleration M1

$v = 3t^2 + 2t + 4$

$20 = 3T^2 + 2T + 4$

$3T^2 + 2T - 16 = 0$ A1 , etc.

Alternative (non-calculus): Attempt at finding the area under force-time graph for impulse $\dfrac{1+(3T+1)}{2}\times T = 0.5(20) - 0.5(4)$	OE	M1

Q	Solution	Mark	Total	Comment
4 (a)	$\boldsymbol{v}_A = \dfrac{(-\mathbf{i}+3\mathbf{j})-(\mathbf{i}+2\mathbf{j})}{\frac{1}{2}} = -4\mathbf{i}+2\mathbf{j}$ $\boldsymbol{v}_B = \dfrac{(2\mathbf{i}-\mathbf{j})-(-\mathbf{i}+\mathbf{j})}{\frac{1}{2}} = 6\mathbf{i}-4\mathbf{j}$	M1 A1		M1 for a difference of two corresponding position vectors divided by $\frac{1}{2}$, A1 for all correct
	${}_A\boldsymbol{v}_B = (-4\mathbf{i}+2\mathbf{j})-(6\mathbf{i}-4\mathbf{j})$	m1		Accept any difference of their velocities
	${}_A\boldsymbol{v}_B = -10\mathbf{i}+6\mathbf{j}$	A1	4	NMS scores 0/4
(b)	$\mathbf{r_0} = (\mathbf{i}+2\mathbf{j})-(-\mathbf{i}+\mathbf{j})$	B1		
	$\mathbf{r} = (\mathbf{i}+2\mathbf{j})-(-\mathbf{i}+\mathbf{j})+(-10\mathbf{i}+6\mathbf{j})t$	M1		M1 for using $\mathbf{r} = \mathbf{r}_0 + {}_A\boldsymbol{v}_B t$ with their ${}_A\boldsymbol{v}_B$.
	$\mathbf{r} = (2-10t)\mathbf{i}+(1+6t)\mathbf{j}$	A1	3	AG
(c)	$AB^2 = (2-10t)^2+(1+6t)^2$	M1		Or $AB=\sqrt{(2-10t)^2+(1+6t)^2}$
	A and B are closest when $\dfrac{\mathrm{d}AB^2}{\mathrm{d}t}\left(\text{or } \dfrac{\mathrm{d}AB}{\mathrm{d}t}\right)=0$	B1		PI
	$\dfrac{\mathrm{d}AB^2}{\mathrm{d}t} = 2(2-10t)(-10)+2(1+6t)6=0$	m1 A1		Condone one sign error **or** one coefficient error All correct
	$t=\dfrac{7}{68}$ or 0.103	A1	5	At least 3 s.f. required if decimal. Accept equivalent fractions
(d)	$AB=\sqrt{(2-10\times 0.103)^2+(1+6\times 0.103)^2}$ **or** $\sqrt{\left(\frac{33}{34}\right)^2+\left(\frac{55}{34}\right)^2}$	m1		Dependent on M1 and m1 in part (c)
	$AB = 1.89$ or $1.886...$	A1	2	At least 3 s.f. required
	Total		**14**	

4 (c) *Alternative 1*:

$AB^2 = (2-10t)^2 + (1+6t)^2$ M1

$AB^2 = 4 - 40t + 100t^2 + 1 + 12t + 36t^2$ A1

A and B are closest when $\frac{dAB^2}{dt}\left(\text{or } \frac{dAB}{dt}\right) = 0$ B1

$-40 + 200t + 12 + 72t = 0$ m1

$t = \frac{7}{68}$ or 0.103 A1

4 (c) *Alternative 2*:

$AB^2 = (2-10t)^2 + (1+6t)^2$ M1

$AB^2 = 4 - 40t + 100t^2 + 1 + 12t + 36t^2$ A1

$AB^2 = 136t^2 - 28t + 5$

$AB^2 = 136\left(\left(t - \frac{7}{68}\right)^2 + \ldots\right)$ m1 A1 m1 for attempt at completing the square of <u>their</u> quadratic

$t = \frac{7}{68}$ or 0.103 A1

4(c) *Alternative 3 (Not in the specification):*

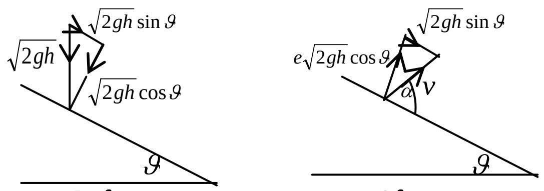
$[(2-10t)\mathbf{i} + (1+6t)\mathbf{j}].[-10\mathbf{i} + 6\mathbf{j}]\ (=0)$ M1 for the scalar product of the r with <u>their</u> ${}_A v_B$
A1 for all correct

$-20 + 100t + 6 + 36t \quad (=0)$ A1

$-20 + 100t + 6 + 36t = 0$ m1 for correctly solving <u>their</u> equation

$t = \frac{7}{68}$ or 0.103 A1

Q	Solution	Mark	Total	Comment
5 (a)	‘No change’ with an attempt to explain	B1		
	Explanation referring to smoothness or lack of friction parallel to the plane	B1	2	
(b)				



Before After

Q	Solution	Mark	Total	Comment
	Speed before impact $= \sqrt{2gh}$ PI	M1		
		A1		Allow $\pm$ expressions
	Parrallel component after impact $= \sqrt{2gh}\sin\vartheta$	A1		
	Perpendicular component after impact $= e\sqrt{2gh}\cos\vartheta$		3	
(c)		M1 A1		Allow M1 for using $\sin\vartheta$ instead of $\cos\vartheta^*$ and + instead of –
	At B,) $0 = \mathrm{e}\sqrt{2gh}\cos\vartheta^* \ t - \frac{1}{2} g\cos\vartheta\, t^2$			
		A1		
	$t = \frac{2\mathrm{e}\sqrt{2gh}\cos\vartheta}{g\cos\vartheta}$ or $\frac{2\mathrm{e}\sqrt{2gh}}{g}$	M1 A1		Allow M1 for using $\cos\vartheta$ instead of $\sin\vartheta^*$ and – instead of +
	$x = \sqrt{2gh}\sin\vartheta^* \ t + \frac{1}{2} g\sin\vartheta\, t^2$			
		m1		Elimination of t. OE
	$AB = \frac{\sqrt{2gh}\sin\vartheta 2\mathrm{e}\sqrt{2gh}}{g} + \frac{g\sin\vartheta 4\mathrm{e}^2 2gh}{2g^2}$			
	$AB = \frac{4gh\mathrm{e}\sin\vartheta}{g} + \frac{8g^2h\mathrm{e}^2\sin\vartheta}{2g^2}$ $AB = 4h\mathrm{e}\sin\vartheta + 4h\mathrm{e}^2\sin\vartheta$ $AB = 4h\mathrm{e}(\mathrm{e}+1)\sin\vartheta$	A1	7	AG, must be convinced
	Total		**12**	

(a) The minimum statement for 2 marks is: ‘No friction, so no change to velocity parallel to the plane’

Allow numerical value of 9.8 for g in part (c), but deduct one A1 mark in part (b) if they have used numerical value.

5(c) *Alternative*	
(At B,) $0 = v\sin\alpha\, t - \frac{1}{2}gt^2\cos\vartheta$	M1
$t = \dfrac{2v\sin\alpha}{g\cos\vartheta}$	m1
$x = v\cos\alpha t + \frac{1}{2}gt^2\sin\vartheta$	M1
$AB = v\cos\alpha\left(\dfrac{2v\sin\alpha}{g\cos\vartheta}\right) + \dfrac{1}{2}g\left(\dfrac{2v\sin\alpha}{g\cos\vartheta}\right)^2\sin\vartheta$	A1
$AB = \dfrac{2v^2\sin\alpha\cos\alpha}{g\cos\vartheta} + \dfrac{2v^2\sin^2\alpha\sin\vartheta}{g\cos^2\vartheta}$	
$\left.\begin{array}{l}\sin\alpha = \dfrac{\sqrt{2gh}\,e\cos\vartheta}{v}\\ \cos\alpha = \dfrac{\sqrt{2gh}\sin\vartheta}{v}\end{array}\right\}$	B1 (for both)
$AB = \dfrac{2v^2 \times \dfrac{\sqrt{2gh}\,e\cos\vartheta}{v} \times \dfrac{\sqrt{2gh}\sin\vartheta}{v}}{g\cos\vartheta} + \dfrac{2v^2\left(\dfrac{\sqrt{2gh}\,e\cos\vartheta}{v}\right)^2\sin\vartheta}{g\cos^2\vartheta}$	m1
$AB = 4he\sin\vartheta + 4he^2\sin\vartheta$	
$AB = 4he(e+1)\sin\vartheta$	A1 AG, must be convinced

Q	Solution	Mark	Total	Comment
6 (a)	Conservation of linear momentum along the line of centres:			
	$2\times 3\cos 60^\circ - 4\times 5\cos 60^\circ = 2\times v$	M1 A1		Condone sign errors Correct with $2v$ or $-2v$
	$v = -3.5$	A1		Or $\frac{7}{2}$, accept 3.5 from consistent working
	Velocity of $A \perp$ to line of centres: $3\sin 60^\circ$	B1		Possibly seen on a diagram
	$V = \sqrt{(3.5)^2 + (3\sin 60^\circ)^2}$	M1		FT their v from above
	$V = 4.36$ or $\sqrt{19}$ ms^{-1}	A1	6	AWRT 4.36, condone missing units
(b)	$\tan^{-1}\frac{3\sin 60^\circ}{3.5}$ *	M1		For correct expression, FT their v from part (a)
	$= 37^\circ$	A1	2	CAO
(c)	$e = \frac{3.5}{3\cos 60^\circ + 5\cos 60^\circ}$	M1		For correct expression, FT their v from part (a)
	$e = 0.875$ or $\frac{7}{8}$	A1	2	CAO
(d)	$I = 4\times 5\cos 60^\circ - 4\times 0$ or $2\times 3\cos 60^\circ - -2\times 3.5$	M1		OE, condone the missing zero term, **FT**
	$I = 10$ Ns	A1	2	CAO, condone missing units
	Total		**12**	

(b) * or $\sin^{-1}\frac{3\sin 60^\circ}{4.36}$ or $\cos^{-1}\frac{3.5}{4.36}$

Q	Solution	Mark	Total	Comment
7 (a)	$J = 2m(2u) - 2m(0)$ $= 4mu$	M1 A1	2	A0 for sign error or $-4mu$ as answer
(b)	$2m(2u) = 2mv_A + mv_B$ $4u = 2v_A + v_B$	M1		CLM
	$\dfrac{v_B - v_A}{2u - 0} = \dfrac{2}{3}$ $4u = 3v_B - 3v_A$	M1 A1		Restitution, condone sign error All correct
	$v_A = \dfrac{8}{9}u$	A1		
	$v_B = \dfrac{20}{9}u$	A1	5	
(c)	$t = \dfrac{s - r}{\frac{20u}{9}}$ or $\dfrac{9(s-r)}{20u}$	M1		$(s-r)$ divided by their v_B from (b)
	Distance travelled by A is $\dfrac{8u}{9} \times \dfrac{9(s-r)}{20u}$	m1		Their $v_A \times$ their time from the line above
	$= \dfrac{2(s-r)}{5}$	A1		OE
	Distance of centre of A from the wall is $s + 2r - \dfrac{2(s-r)}{5} = \dfrac{3s + 12r}{5}$	A1	4	AG
(d)	$w_B = \dfrac{20u}{9} \times \dfrac{2}{5}$	M1		Their v_B from (b) $\times \dfrac{2}{5}$
	$= \dfrac{8}{9}u$	A1		
	A and B have the same speed $\Rightarrow$ The distance between them will be halved to $\dfrac{1}{2}\left(\dfrac{3s+12r}{5} - 3r\right)$ or $\dfrac{3s - 3r}{10}$	M1		Explanation not needed
	$\therefore$ The required distance is $\dfrac{1}{2}\left(\dfrac{3s+12r}{5} - 3r\right) + r = \dfrac{3s + 7r}{10}$	A1	4	Simplification not required
	Total		**15**	

(a) Condone omission of $-2m(0)$.

7(d)	*Alternative1*:	
	$w_B = \frac{20u}{9} \times \frac{2}{5}$	M1
	$= \frac{8}{9}u$	A1
	Time taken by B to collide again $= \frac{x}{\frac{8}{9}u}$	
	Time taken by A to collide again $= \frac{\frac{3s+12r}{5} - 3r - x}{\frac{8}{9}u}$	
	$x = \frac{3s+12r}{5} - 3r - x$ or $\frac{3s-3r}{10}$	
	The distance of the centre of B from the wall $= \frac{3s-3r}{10} + r = \frac{3s+7r}{10}$	A1
	Alternative 2:	
	$w_B = \frac{20u}{9} \times \frac{2}{5}$ M1	
	$= \frac{8}{9}u$ A1	
	Velocity of A relative to B $= \frac{16u}{9}$	
	Distance to collision $= \frac{3s+12r}{5} - 3r$	
	Time to collision $= \frac{\frac{3s+12r}{5} - 3r}{\frac{16u}{9}}$	
	$= \frac{27s-27r}{80u}$	
	Distance moved by B $= \frac{8u}{9}\left(\frac{27s-27r}{80u}\right)$ M1	
	The required distance $= \frac{8u}{9}\left(\frac{27s-27r}{80u}\right) + r = \frac{3s+7r}{10}$ A1	

A-level

Mathematics

MM03

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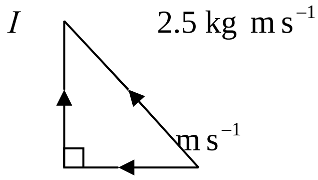
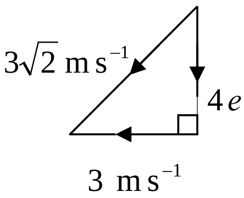
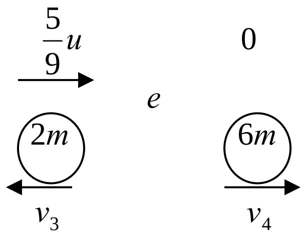
Question	Solution	Marks	Total	Comments
1	$[F] = \mathrm{MLT}^{-2}$	B1		B1: Correct dimensions of F
	$\mathrm{MLT}^{-2} = \left(\mathrm{LT}^{-1}\right)^{\alpha}\left(\mathrm{L}^{2}\right)^{\beta}\left(\mathrm{ML}^{-3}\right)^{\gamma}$	M1		M1: Substituting the dimensions of the quantities into the given equation to obtain RHS correctly.
	$= \mathrm{M}^{\gamma}\mathrm{L}^{\alpha+2\beta-3\gamma}\mathrm{T}^{-\alpha}$	m1		m1: Collecting indices on RHS. Could be implied by later work.
	$\left.\begin{array}{l}\gamma = 1\\ \alpha + 2\beta - 3\gamma = 1\\ -\alpha = -2\end{array}\right\}$	A1 m1	6	A1: $\gamma = 1$ m1: Two correct equations for α and β.
	$\alpha = 2$, $\beta = 1$	A1		A1: Correct values for α and β. Condone use of units instead of dimensions.
	Total		**6**	

Question	Solution	Marks	Total	Comments
2 (a)	$x = u\cos\alpha t$	M1		M1: Correct expression for horizontal displacement.
	$t = \dfrac{x}{u\cos\alpha}$	A1		A1: Correct expression for t.
	$y = u\sin\alpha t - \dfrac{1}{2}gt^2$			
	$y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \dfrac{1}{2}(9.8)\left(\dfrac{x}{u\cos\alpha}\right)^2$	M1		M1: Correct expression for vertical displacement. Allow sign errors.
		m1		m1: Elimination of t from equation for vertical displacement.
	$y = x\tan\alpha - \dfrac{4.9x^2}{u^2\cos^2\alpha}$ **AG**	A1	5	A1: Correct result from correct working. Penalise use of g = 9.81.
(b)(i)	$-s = s\tan 55^\circ - \dfrac{4.9s^2}{21^2\cos^2 55^\circ}$	M1		M1: Substituting $\pm s$ for x and y.
	$s = \dfrac{(1+\tan 55^\circ)21^2\cos^2 55^\circ}{4.9}$	m1		m1: Making s the subject of their equation.
	$s = 71.9$	A1	3	A1: AWRT 71.9 Condone use of g = 9.81 which gives 71.8.
(ii)	$\dot{x} = 21\cos 55^\circ = 12.045$	B1		B1: Correct expression or value for horizontal component of velocity.
	$\dot{y} = 21\sin 55^\circ - 9.8 \times \dfrac{71.895}{21\cos 55^\circ}$ or $\dot{y}^2 = (21\sin 55^\circ)^2 - 2(9.8)(-71.895)$	M1		M1: Correct expression or value for vertical component of velocity, with their answer to (b)(i).
	$\dot{y} = -41.292$	A1		A1: Correct expression or value.
	$\tan^{-1}\dfrac{-41.292}{21\cos 55^\circ}$	m1		m1: Use of tan with their velocity components.
	$= -74^\circ$ or 74°	A1	5	A1: Correct angle to nearest degree. CAO.
	Total		**13**	

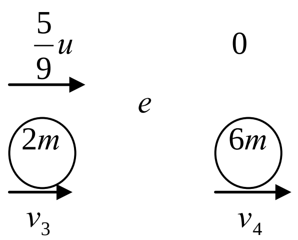
(b)(ii)	Alternative: $y = x\tan\alpha - \dfrac{4.9x^2}{u^2\cos^2\alpha}$ $\dfrac{dy}{dx} = \tan\alpha - \dfrac{2(4.9)x}{u^2\cos^2\alpha}$ $= \tan 55^\circ - \dfrac{2(4.9)(71.895)}{21^2 \times \cos^2 55^\circ}$ $= -3.428$ The angle $= \tan^{-1}(-3.428)$ $= -74^\circ$ or 74°	B1 M1 A1 m1 A1	 5	B1: Correct derivative. M1: Substituting values. A1: Correct value of the derivative m1: Use of tan to find the angle. A1: Correct angle to nearest degree. CAO.

Question	Solution	Marks	Total	Comments
3 (a)		B1		B1: Momentum – Impulse triangle with right angle. Can be implied by a correct equation.
	$2.5^2 = 1.5^2 + I^2$	M1		M1: Use of Pythagoras to obtain a correct equation. OE for example $5^2 = 3^2 + \left(\frac{I}{0.5}\right)^2$
	$I = 2$ N s	A1	3	A1: Correct impulse.
(b)	After the impact:	B1		B1: Sight of perpendicular component as $4e$. Could be implied by a correct equation.
		B1		B1: Correct velocity diagram, PI by a correct equation
	$\left(3\sqrt{2}\right)^2 = (4e)^2 + 3^2$	M1		M1: Use of Pythagoras to obtain a correct equation.
	$e = \frac{3}{4}$ or 0.75	A1	4	A1: Correct coefficient of restitution.
	Total		**7**	

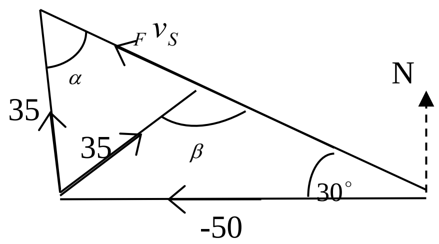
(a)	Alternative: $I\mathbf{j} = 0.5(5\cos\alpha\mathbf{i} + 5\sin\alpha\mathbf{j}) - 0.5(3\mathbf{i})$ $2.5\cos\alpha - 1.5 = 0$ $\cos\alpha = 0.6$ $\sin\alpha = 0.8$ $I = 0.5(5 \times 0.8)$ $I = 2$	B1 M1 A1	 3	B1: Correct vector equation. M1: Correct value for $\sin\alpha$. A1: Correct impulse.
(b)	Alternative: $3 = 3\sqrt{2}\sin\beta$ $\cos\beta = \frac{1}{\sqrt{2}}$ $e = \frac{3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)}{\frac{2}{0.5}}$ $e = \frac{3}{4}$ or 0.75	B1 B1 M1 A1	 4	B1: Correct equation for motion parallel t B1: Value for $\cos\beta$ or $\beta = 45°$. M1: Correct expression for e or correct eq A1: Correct impulse.

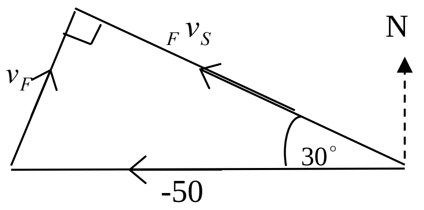
Question	Solution	Marks	Total	Comments
4 (a) (i)	$mu = mv_1 + 2mv_2$ OE	M1 A1		M1: Equation with three momentum terms. A1: Correct equation.
	$u = v_1 + 2v_2$			
(ii)	$\frac{2}{3}u = v_2 - v_1$ OE	M1 A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$3v_2 = \frac{5}{3}u$			
(b)	$v_2 = \frac{5}{9}u$ **AG**	A1	6	A1: Correct speed of B, from correct working.
	$v_1 = u - \frac{10}{9}u$			
	$v_1 = -\frac{1}{9}u$			
	The speed of A is $\frac{1}{9}u$	A1		A1: Correct speed of A. Do not accept negative speed
				
	$2m\left(\frac{5}{9}u\right) = -2mv_3 + 6mv_4$ OE	M1 A1		M1: Equation with three momentum terms. A1: Correct equation
	$\frac{10}{9}u = -2v_3 + 6v_4$	M1 A1		
	$e\left(\frac{5}{9}u\right) = v_3 + v_4$ OE			M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation
(c)	$\frac{10}{9}u = -2v_3 + 6\left(\frac{5}{9}ue - v_3\right)$	m1		
	$8v_3 = \frac{10}{3}ue - \frac{10}{9}u$	A1F	8	m1: Solving equations to find the speed of B after the second collision.
	$v_3 = \frac{5}{12}ue - \frac{5}{36}u$ OE			A1F: Correct speed of B after the second collision. FT their equations
			2	

Q	Solution	Marks	Total	Comments
	second collision $\Rightarrow$ $\frac{5}{12}ue - \frac{5}{36}u > \frac{1}{9}u$ $\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5}$ or 0.6	M1 A1F		M1: For the inequality $v_3 > v_1$ A1F: Correct value of k. FT their $v_3 > v_1$. The value of k must be less than 1 and greater than 0 to score A1F
	Equal radii $\Rightarrow$ Velocities are parallel to the line of centr	B1 B1		B1: Comment about equal radii or same size. B1: Comment about the line of centres.
	Total		**16**	

	(b) Alternative:			
				
	$2m\left(\frac{5}{9}u\right) = 2mv_3 + 6mv_4$ $\frac{10}{9}u = 2v_3 + 6v_4$	M1A1		M1: Equation with three momentum terms. A1: Correct equation.
	$e\left(\frac{5}{9}u\right) = v_4 - v_3$ $\frac{10}{9}u = 2v_3 + 6\left(\frac{5}{9}ue + v_3\right)$	M1A1		M1: Newton's Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$8v_3 = \frac{10}{9}u - \frac{10}{3}ue$ $v_3 = \frac{5}{36}u - \frac{5}{12}ue$ OE	m1A1F		m1: Solving equations to find the velocity of *B* after the second collision. A1F: Correct velocity of *B* after the second collision. FT their equations.
	second collision $\Rightarrow$ $\frac{5}{36}u - \frac{5}{12}ue < -\frac{1}{9}u$	M1		M1: For the inequality $v_3 < v_1$
	$\frac{5}{12}ue > \frac{9}{36}u$ $e > \frac{3}{5}$ or 0.6	A1F		A1F: Correct value of *k*. The value of *k* must be less than 1 and greater than 0 to score A1F

Question	Solution	Marks	Total	Comments
5	$\cos\alpha = \frac{3}{5}$ or 0.6 and $\cos\beta = \frac{5}{13}$ or 0.3846...	B1		B1: Correct values for $\cos\alpha$ and $\cos\beta$.
	$2(4\cos\alpha)+1(2.6\cos\beta)=2v_A+1v_B$ $2(2.4)+1(1)=2v_A+1v_B$	M1A1		M1: Four term momentum equation along the line of centres. A1: Correct equation. May be in terms of α and β.
	$\frac{4}{7}(4\cos\alpha-2.6\cos\beta)=v_B-v_A$ $\frac{4}{7}(2.4-1) = v_B - v_A$ $\begin{cases} 5.8 = 2v_A + v_B \\ 0.8 = v_B - v_A \end{cases}$	M1 A1		M1: Newton’s Law of Restitution. (Allow sign errors.) A1: Correct equation.
	$v_A = \frac{5}{3}$ ms^{-1}	A1	11	A1: Correct velocity of A. AWRT 1.67
	$v_B = \frac{37}{15}$ ms^{-1}	A1		A1: Correct velocity of B. AWRT 2.47
	$V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (4\sin\alpha)^2}$	m1		m1: Finding speed of A with their v_A. May be in terms of α and β.
	$V_A = \sqrt{\left(\frac{5}{3}\right)^2 + (3.2)^2} = 3.61$ ms^{-1}	A1		A1: Correct speed. AWRT 3.61
	$V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.6\sin\beta)^2}$	m1		m1: Finding speed of B with their v_B. May be in terms of α and β.
	$V_B = \sqrt{\left(\frac{37}{15}\right)^2 + (2.4)^2} = 3.44$ ms^{-1}	A1		A1: Correct speed. AWRT 3.44
	Total		**11**	

6 (a)(i)		B1 B1		B1: For one velocity triangle, could be implied by later working. B1: For the other velocity triangle drawn together or separately, could be implied by the correct 2nd angle
(ii)	$\frac{\sin\alpha}{50}=\frac{\sin 30^\circ}{35}$ or $\frac{\sin\beta}{50}=\frac{\sin 30^\circ}{35}$ $\left.\begin{array}{l}\alpha = 45.58^\circ\\ \beta = 134.42^\circ\end{array}\right\}$ Bearings: $\left.\begin{array}{l}346^\circ\\ 074^\circ\end{array}\right\}$	M1 A1 A1	5	M1: Correct use of sine rule to find α or β. A1: **Either** angle correct. A1: **Two** correct bearings. Accept 74°.
(b)	Angle for shorter time: 45.58° $\frac{_F v_S}{\sin 104.42^\circ}=\frac{35}{\sin 30^\circ}$ $_F v_S = 67.79 \quad \text{km h}^{-1}$ $t=\frac{8}{67.79}$ $= 0.118 \text{ h} \quad \text{or} \quad 7.08 \text{ min}$	B1 M1 A1 m1 A1F	5	B1: Selecting the smaller of their two angles from part (a). M1: Using the sine rule to find the speed of the frigate relative to the ship, with their angle. A1: Correct speed. m1: Using distance over speed. A1F: Correct time. FT their speed. Full marks can be scored by using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.

	 $v_F = 50\sin 30^\circ$ OE $v_F = 25$ kmh^{-1}	B1 M1 A1	3	B1: Correct right angled velocity triangle. Could be implied by later working. M1: Use of trigonometry to find speed. A1: Correct speed. CAO.
	Total		**13**	

(a)(ii)	Alternative: Angle for shorter time : 45.58° $t(50\cos 30^\circ + 35\cos 45.58^\circ) = 8$ $\left(t = \dfrac{8}{50\cos 30^\circ + 35\cos 45.58^\circ}\right)$ $t = 0.118$ h or 7.08 min	B1 M1A1 m1 A1F	 5	B1: Selecting the smaller of their two angles from part (a). M1: For $50\cos 30^\circ \pm 35\cos 46^\circ$ A1: Correct expression. m1: Using distance over speed. A1F: Correct time. FT their angle. Full marks can be scored by using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.
	Alternative: Angle for shorter time : 45.58° $\dfrac{d}{\sin 30^\circ} = \dfrac{8}{\sin 104.42^\circ}$ $d = 4.130$ km $\left(t = \dfrac{4.130}{35}\right)$ $t = 0.118$ h or 7.08 min	B1 M1 A1 m1 A1F	 5	B1: Selecting the smaller of their two angles from part (a). M1: Using the sine rule to find the distance travelled by the frigate with their angle. A1: Correct distance m1: Using distance over speed. A1: Correct time. FT their angle. Full marks can be scored b using both angles **and** choosing the shorter time. If both times calculated and none selected do not award final A1 mark.

Question	Solution	Marks	Total	Comments
7 (a)	$y = u\sin(\alpha - \vartheta)t - \frac{1}{2}g\cos\vartheta t^2$	M1		M1: Expression for perpendicular height of particle above the plane. Accept wrong angles for M1 but **not** sin and cos in wrong places.
	$0 = u\sin(\alpha - \vartheta)t - \frac{1}{2}g\cos\vartheta t^2$	A1		A1: Correct expression with $y = 0$.
	$t = \frac{2u\sin(\alpha - \vartheta)}{g\cos\vartheta}$	m1 A1	4	m1: Solving for non-zero t. A1: Correct t.
(b)	$u\sin\alpha - gt = 0$ $t = \frac{u\sin\alpha}{g}$	M1		M1: Velocity equation to find time to A.
	$\frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha - \vartheta)}{g\cos\vartheta}$	A1		A1: Correct time.
	$\sin\alpha\cos\vartheta = 2\sin(\alpha - \vartheta)$ $\sin\alpha\cos\vartheta = 2\sin\alpha\cos\vartheta - 2\cos\alpha\sin\vartheta$	m1		m1: Forming an equation using their time from part (a) and this time.
	$\left.\begin{array}{l}\sin\alpha\cos\vartheta = 2\cos\alpha\sin\vartheta \\ \frac{\sin\alpha}{\cos\alpha} = 2\frac{\sin\vartheta}{\cos\vartheta}\end{array}\right\}$	M1	5	M1: Use of identity to eliminate compound expressions. It is not enough to only expand $\sin(\alpha - \theta)$ in the expression in part (a) without anything else.
	$\tan\alpha = 2\tan\vartheta$	A1		A1: Seeing required expression derived with $k = 2$.
	Total		**9**	
	TOTAL		**75**	

(b)	Alternative: Taking x and y axes parallel and perpendicular to the plane respectively and using $\tan\theta = \frac{-\dot{y}}{\dot{x}}$ or equivalent,			
	$\left(u\cos(\alpha-\theta) - g\frac{2u\sin(\alpha-\theta)}{g\cos\theta}\sin\theta\right)\tan\theta = -u\sin(\alpha-\theta) + \frac{g2u\sin(\alpha-\theta)}{g\cos\theta}\cos\theta$	M1 A1		M1: Correct terms, allow sign errors. A1: All correct
	$\cos(\alpha-\theta)\tan\theta = \sin(\alpha-\theta)(2\tan^2\theta+1)$			
	$(\cos\alpha\cos\theta+\sin\alpha\sin\theta)\tan\theta = (\sin\alpha\cos\theta-\sin\theta\cos\alpha)(2\tan^2\theta+1)$	M1		M1: Use of identities to eliminate compound expressions.
	$\tan\alpha\tan^2\theta+\tan\alpha-2\tan^3\theta-2\tan\theta=0$			
	$\tan\alpha(1+\tan^2\theta) = 2\tan\theta(1+\tan^2\theta)$	m1		m1: Rearranging to the required form.
	$\tan\alpha = 2\tan\theta$	A1	5	A1: Seeing required expression derived with $k = 2$.